Information quality and regime change: Evidence from the lab*

Leif Helland[†] Felipe S. Iachan[‡]

Ragnar E. Juelsrud[§] Plamen T. Nenov[¶]

Abstract

We experimentally test the effects of information quality on regime stability in a global game of regime change. The game features a payoff structure such that more dispersed private information induces agents to attack more and reduces regime stability in the Bayesian Nash Equilibrium. We show that subjects in the lab do not play as predicted by the theory. Rather, more dispersed information makes subjects more cautious, increasing regime stability. We show that this finding is consistent with a modified global game model in which agents engage in level-k thinking. In the level-k model, information quality affects agents' actions through a novel channel, which can reverse the comparative statics from the fully rational model.

Keywords: global games, information quality, level-k thinking, across-type strategic complementarity, finite mixture model.

JEL Codes: C72, C9, D82, D9

*We would like to thank Frank Heinemann, Nagore Iriberri, Terri Kneeland, participants at the 9th International Conference of the French Society for Experimental Economics and the 2018 ESA World Meeting for useful comments and suggestions. This research was financed by the Research Council of Norway, grant 250506 and, in part, by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. This working paper should not be reported as representing the views of Norges Bank. The views expressed are those of the author and do not necessarily reflect those of Norges Bank.

[†]BI Norwegian Business School, e-mail: leif.helland@bi.no@bi.no (corresponding author).

[‡]FGV EPGE, e-mail: felipe.iachan@fgv.br

[§]Norges Bank, e-mail: ragnar@juelsrud.no.

[¶]BI Norwegian Business School, e-mail: plamen.nenov@bi.no.

1 Introduction

Global games of regime change are commonly used to analyze important economic phenomena involving elements of coordination, such as currency crises, bank runs, and political change.[1] A central question in this literature – both from a theoretical and an applied perspective – is how information quality – the precision of agents' private information – affects the probability of a successful coordinated attack.

In this paper, we experimentally test how a change in private information precision affects regime stability in a standard global game of regime change. Specifically, we consider a global game in which agents receive private information with certain dispersion. The payoff structure is such that higher private information dispersion makes agents more likely to attack in equilibrium and, hence, regime stability decreases.[2] Our set-up, therefore, differs from the majority of other experimental evaluations of global games of regime-change, which typically compare the effects of private versus public information.

We let subjects play a series of games where they take binary decisions – attack or not attack. Their payoff from attacking depends both on an underlying state and on the actions of others. If a sufficient number of agents choose to attack (given the value of the underlying state), then all attacking agents obtain a discretely higher payoff relative to not attacking. In addition, the higher the value of the state, the higher the discrete payoff from a successful attack.[3] Finally, agents obtain private signals about the underlying state with some dispersion. We set up the payoffs of agents to correspond to the literature on speculative currency attacks, where theory predicts that more dispersed information is destabilizing (Heinemann and Illing, 2002, Iachan and Nenov, 2015). In this setting, we

[1]See Morris and Shin, 1998 for currency crises, Rochet and Vives (2004) and Goldstein and Pauzner (2005) for bank runs, and Edmond (2013) for political change.

[2]In general, the effect of a change in private information dispersion on regime stability is ambiguous and depends on the payoff structure that is generated by the underlying economic environment (Iachan and Nenov, 2015). To provide a clear theoretical prediction to test in a laboratory setting, we opt for a specific payoff structure that leads to the aforementioned comparative static.

[3]Therefore, a higher state in our abstract game can be interpreted as a lower value of a common economic fundamental.

compare subjects' behavior in two treatments, one where private information dispersion is low ("Low Noise treatment") and one where the information dispersion is high ("High Noise treatment").

Our experimental results run counter to the baseline theoretical predictions. Focusing on the differences in average estimated strategic cutoffs, we find that they are significantly *lower* in the Low Noise treatment compared to the High Noise treatment. Therefore, the comparative statics are the opposite of what the theory predicts. More dispersed information makes agents less, not more, aggressive.

Motivated by this finding, we modify the standard global game by assuming that players have limited depths of reasoning. In particular, we focus on one specific non-equilibrium theory that has received recent experimental and theoretical attention in the literature on global games and informational frictions (Kneeland, 2016, Angeletos and Lian, 2017), namely level-k thinking (Nagel, 1995; Stahl and Wilson, 1995).[4] Models of level-k thinking assume that agents have limited depths of reasoning and, at the same time, provide a specific structure to agents' beliefs. We show that this departure from fully rational play can significantly alter the prediction of the standard global games theory. Specifically, with level-k types, the effect of more dispersed information on agents' actions and regime stability can be reversed relative to the game with fully rational players.

Intuitively, with level-k agents, different cognitive types have different strategic cutoffs – the value of private signals above which agents are better off attacking. At the same time, there is a strategic complementarity across level-k types, so that the aggressiveness of a lower cognitive type influences the aggressiveness of higher cognitive types. Higher information dispersion acts to *attenuate* this across-type strategic complementarity. Specifically, when lower cognitive types are relatively more aggressive than the higher cognitive types, the decrease in coordination due to more dispersed private information stabilizes the regime, as perceived by the higher cognitive types, which reduces those agents' willingness to attack.

[4] An overview of models and evidence of non-equilibrium strategic thinking is provided in Crawford, Costa-Gomes, and Iriberri (2013).

Put differently, higher cognitive types become *less* aggressive with higher information dispersion. If this effect is strong enough, then increased information dispersion is *stabilizing* in the level-k model rather than destabilizing.

To assess whether such a level-k model is quantitatively consistent with our experimental findings, we follow Kneeland (2016) and structurally estimate a finite mixture model of play with different level-k types, using data from both treatments. We additionally assume that players are risk-averse, which ensures that the *levels* of the strategic cutoffs of behavioral types are in line with our empirical data.[5] We estimate a common distribution of level-k types for *both* treatments, assuming fixed play by $L0$ types *across* the treatments. Therefore, our results are not driven by variation in $L0$ types' perceived play or different distributions of the level-k types across the two treatments but are purely due to the effect of information dispersion on actions. Our estimates show that a level-k model, augmented with risk-averse players, can explain the observed differences in the strategic cutoffs estimated from the two treatments.

Related literature

Initial coordination experiments focused on static games with complete information (Cooper, DeJong, Forsythe, and Ross, 1990, 1992; Straub, 1995; Van Huyck, Battalio, and Beil, 1990). Such games have multiple equilibria and strategic uncertainty comes to the forefront. As a response to this indeterminacy, the theory of global games was developed by Carlsson and van Damme (1993). The theory was later advanced by Morris and Shin (1998) to macroeconomic applications. The global games framework provides an explicit model of strategic uncertainty. It shows that coordination games with multiple equilibria under complete information may have a unique equilibrium if certain parameters of the payoff function are private information instead of common knowledge.

[5] Risk aversion on its own cannot reverse the comparative statics in a standard global game model, since it only dampens the effect of information dispersion in that model.

Heinemann, Nagel, and Ockenfels (2004) is the experimental paper closest to ours. It tests the predictions of the theory of global games in a setting where the net payoff from a successful attack is increasing in the underlying fundamental. In the unique global games equilibrium agents use monotone cutoff strategies. Heinemann, Nagel, and Ockenfels (2004) document that subjects tend to use such strategies, both under public and private information. Widespread use of monotone, or near monotone, cutoff strategies has also been documented in a broader class of global games experiments (Cornand and Heinemann, 2014; Avoyan, 2017; Szkup and Trevino, 2017).[6] Our experimental results are in line with these findings.

Kneeland (2016) analyses global games where agents engage in level-k thinking.[7] She theoretically characterizes a coordination game where agents are of different level-k types. Using experimental data from Heinemann, Nagel, and Ockenfels (2004), she shows that the level-k model fits the data better than the fully rational model. Relative to this paper, we provide a novel prediction on the effect of changes in information dispersion on different cognitive types' strategic cutoffs, and show experimental support for this prediction.

When the net payoff of a successful attack is increasing in the fundamentals as in our experimental game, more precise private information should induce agents to become less aggressive in equilibrium. The experimental literature, however, seems to document the opposite. Heinemann, Nagel, and Ockenfels (2004) compare behavior under complete information to behavior under incomplete information. They find that subjects behave more cautiously under incomplete information. Cabrales, Nagel, and Armenter (2007) test the global games theory in a series of two-person games with a simplified information structure. The design ensures that equilibrium is reached after only four rounds of elimination of (interim) strictly dominated strategies. They find that subjects converge to the unique global

[6] Heinemann, Nagel, and Ockenfels (2009) develop a method to measure strategic uncertainty as an alternative to varying the parameters of the game exogenously. They also find widespread use of cutoff strategies. Heggedal, Helland, and Joslin (2018) find widespread use of cutoff strategies in a coordination game with type uncertainty rather than uncertainty about fundamentals.

[7] Cornand and Heinemann (2014) analyze the relative weighting of public and private signals in a global game by a k-level model and a cognitive hierarchy model.

games equilibrium under incomplete information, but that subjects behave less cautiously under complete information.

In a recent contribution, Szkup and Trevino (2017) also consider an experimental setting where the precision of private signals varies across treatments. Like us, they find that the comparative statics are reversed relative to what the theory predicts. To explain the reversal in the theoretical comparative statics Szkup and Trevino (2017) argue that there is a link between players' perception of strategic uncertainty and fundamental uncertainty. They propose a "sentiment theory", where as fundamental uncertainty increases, players also become more pessimistic about the actions of others, which is modeled via a "belief residual" term. When the belief residual term is allowed to depend flexibly on the level of signal precision across treatments, the modified model can account for departures from the theoretical benchmark.

We differ from and complement Szkup and Trevino (2017) along several dimensions. First, we differ in the experimental setting. Specifically, they investigate a two-player investment game similar to Carlsson and van Damme (1993), while we consider a larger coordination game of regime change. Second, and more importantly, we explain the reversed comparative statics with a theory based on bounded rationality and limited depth of reasoning. In that theory we identify a novel effect of information quality on agents' actions, which is absent in the fully rational model. We then evaluate the ability of this theory to explain the data. Importantly, in our empirical investigation of the level-k theory we keep both the distribution of level-k types and the perceived play of L0 types fixed as we change information quality.

2 Theoretical predictions

Set-up

We consider a regime-change game that can serve as a simple representation of the strategic interactions involved in currency crises (Morris and Shin, 1998), debt rollover (Rochet and Vives, 2004, Goldstein and Pauzner, 2005), and political change (Edmond, 2013). We follow the notation from Iachan and Nenov (2015), with a few modifications that are necessary to enable a laboratory-based test of the theoretical predictions relating the information structure to players' actions and regime stability. Most importantly, we assume that there is a discrete number N of players.

Agents take a binary action $s_i \in \{0, 1\}$ simultaneously. We interpret $s_i = 1$ as player i attacking the status quo. We let $Z = \sum_i s_i$ denote the number of agents who choose $s_i = 1$. A state variable Y (the fundamentals) determines agents' payoffs, and also the minimal number of agents required for a successful attack. We assume that Y is distributed uniformly on $[0, M]$, for $M > 0$ and is not directly observed by agents, who hold this distribution as their prior belief about the state.

Regime change occurs if at least a fraction $g(Y) \in (0, 1)$ of agents attack, where $g(\cdot)$ is a *decreasing* function of the fundamentals.[8] We define $G_N(Y) \equiv \lceil g(Y) N \rceil$, so that regime change occurs if, and only if, $Z \geq G_N(Y)$.

We normalize the payoff from action $s_i = 0$ to zero in the case of both regime change and status quo survival and specify the payoffs in case of $s_i = 1$ as follows: the payoff to a player who attacks is $D(Y)$ in case of regime change and $U(Y)$ in case of status quo survival. We assume that $D(Y) > 0$ and $U(Y) < 0$ and that both are either constant or strictly increasing in Y. As a consequence, actions are strategic complements.

Before choosing actions, agents observe noisy signals about the state Y. Specifically, we assume that player i observes a signal $x_i = Y + \eta_i$, where η_i's are distributed uniformly on

[8] Therefore, higher Y means weaker fundamentals in this setting.

$[-\epsilon, \epsilon]$, $\epsilon > 0$, and $\epsilon \ll M$, independently across players. Also, η_i is independent of the realization of Y. We define $E_{x_i}[\cdot]$ as the expectation with respect to the information set of an agent that receives signal x_i.

Equilibrium

The definition of a Bayesian Nash Equilibrium for our game is standard (see Morris and Shin (2003)). We restrict attention to equilibria in monotone strategies. A monotone strategy Y^* is such that $s(x_i) = 1$ iff $x_i > Y^*$. In that case it is straightforward to apply standard results from global games to show that there is a unique equilibrium. Furthermore, the restriction is without loss of generality (Morris and Shin, 2003).

We call the critical value Y^* the *strategic cutoff*. Note that for a given value Y^* in the finite-player case, the number of players who observe a signal above Y^* and thus choose $s_i = 1$ is stochastic. Given a value of the fundamental Y, with signals uniformly distributed on the interval $[Y - \epsilon, Y + \epsilon]$, the probability that at least K players get a signal above Y^* is given by the tail distribution of a Binomial random variable

$$\overline{F}_N(K, Y, Y^*) = \sum_{k \geq K}^{N} \begin{pmatrix} N \\ k \end{pmatrix} p(Y, Y^*, \epsilon)^k (1 - p(Y, Y^*, \epsilon))^{N-k} \tag{1}$$

where

$$p(Y, Y^*, \epsilon) = \min\left\{\max\left\{0, \frac{Y + \epsilon - Y^*}{2\epsilon}\right\}, 1\right\} \tag{2}$$

Therefore, the probability of regime change given a state Y is

$$P(Y, Y^*) \equiv \overline{F}_N(G_N(Y), Y, Y^*) \tag{3}$$

Note that $P(Y, Y^*) = 1$ for $Y \geq Y^* + \epsilon$ and $P(Y, Y^*) = 0$ for $Y \leq Y^* - \epsilon$. Also, $P(Y, Y^*)$ is defined as a tail distribution evaluated at the endogenous $G_N(Y)$. It is also convenient to define the probability of regime change for a player that attacks ($s_i = 1$). We define this probability by $\tilde{P}(Y, Y^*)$. Specifically, a player that attacks expects regime change to occur if at least $G_N - 1$ of the remaining $N - 1$ other players attack, which gives

$$\tilde{P}(Y, Y^*) \equiv \overline{F}_{N-1}(G_N(Y) - 1, Y, Y^*) \tag{4}$$

Given this probability of regime change, Y^* is determined by an indifference condition for a marginal agent – a player who observes a signal $x_i = Y^*$. Specifically, Y^* solves

$$E_{Y^*}\left[D(Y)\tilde{P}(Y, Y^*) + U(Y)\left(1 - \tilde{P}(Y, Y^*)\right)\right] = 0. \tag{5}$$

That is, for a marginal agent, the expected payoff from attacking equals the payoff from not attacking.

As shown by Iachan and Nenov (2015), with a continuum of players, the effect of information quality on the equilibrium of this game depends on a comparison of the sensitivities of payoffs in the case of regime change and status quo survival. In our experiment, we focus on the case where $U(Y) = U < 0$ and $D(Y)$ is strictly increasing in Y. This nests many global games applications, such as the literature on currency crises (Morris and Shin, 1998).

The prediction of the model that we aim to test experimentally is the comparative static of Y^* with respect to ϵ. In this context, increased information dispersion is destabilizing (Iachan and Nenov, 2015). That is, if $N \to \infty$, $U(Y) = U < 0$, $\forall Y$ and $D(Y)$ is strictly increasing, then $\frac{\partial Y^*}{\partial \epsilon} < 0$.[9]

[9] The same comparative statics also hold away from the limit $N \to \infty$.

3 Experimental implementation

In order to implement the model in the lab, we closely follow Heinemann, Nagel, and Ockenfels (2004).[10] The experiment is implemented as a series of 8 independent rounds. In each round each subject makes 10 independent binary choices. We organize subjects in groups of $N = 10$, with subjects indexed by i. The rules of the game are made public knowledge through the reading of instructions aloud.[11] Unique subjects are used in all sessions. The language of the experiment is neutral.

At the beginning of each round, 10 different values of Y are drawn, where Y is distributed uniformly on $[0, 100]$. For any realization of Y, individual signals x_i are then drawn independently according to a uniform distribution on $[Y - \epsilon, Y + \epsilon]$. Each individual signal is revealed to subject i but not to the other subjects in the group. Within a treatment and a given round, the list of fundamentals (Y) are identical for the subjects in different groups, while the list of signals (x_i) varies over subjects. Given their signals, subjects are asked to make a decision, A or B for each of the 10 decision situations in that round. In the context of the model outlined above, A correspond to $s_i = 0$ and B correspond to $s_i = 1$. Subjects get a feedback after each round. For each of the 10 games on the list, this feedback consists of the number Y, the number of subjects that decided for A and B, and the subject's own payoff.

If a subject chooses A, she receives an endowment of 20 experimental currency units. If the subject chooses B, she receives a payoff which depends on both the number of other subjects who chose B and the state Y. Regime change takes place if $G_{10}(Y) = [10\,(80 - Y)\,/60]$ individuals choose B. More specifically, our payoff structure is as follows. Let Z be the number of agents in a group that attacks. $\pi(Y, Z)$ is the net payoff from choosing B, given the fundamental Y and the actions of the group members. $\pi(Y, Z)$ is increasing in Y.

[10] We adopt the same payoff functions and other parameters as in their (T=20; Z=60) treatments. Our experiment is based on the same zTree files and the same instructions as their experiment. The only differences, aside from the subject pool, is that we consider groups of 10 rather than 15 subjects and that we replace their complete information treatment with our High Noise treatment.

[11] Instructions for the high noise treatment are available at: http://www.leifhelland.net/working-papers/

Treatments	Low Noise ($\epsilon = 10$)	High Noise ($\epsilon = 20$)
Theoretical cutoff	$Y_L^* = 41.4$	$Y_H^* = 37.8$
Expected treatment difference	$Y_L^* - Y_H^* \quad =$	3.6

Table 1: Theoretical prediction.

$$\pi(Y, Z) = \begin{cases} Y - 20 & : Z \geq G_{10}(Y) \\ -20 & : Z < G_{10}(Y) \end{cases} \tag{6}$$

With this set-up, observe that playing A is dominant if $Y < 20$ and playing B is dominant if $Y > 74$.

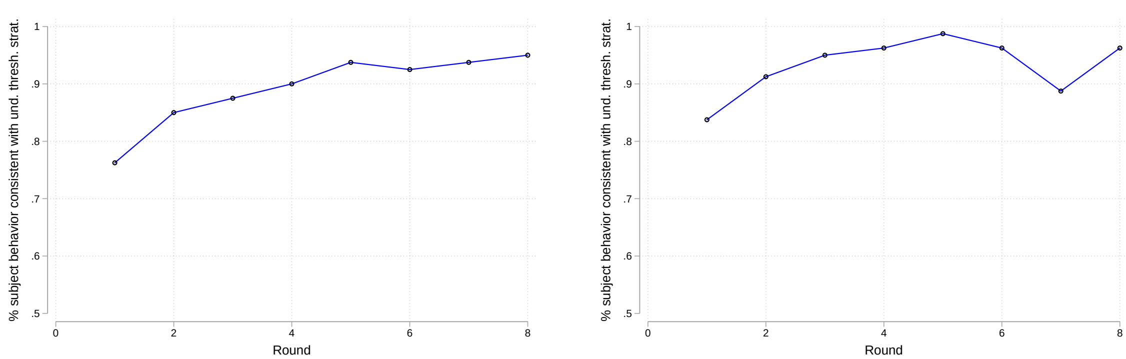
We run a simple design in which the only treatment is the dispersion in the private signals, parametrized by the noise term ϵ. Specifically, we consider two treatments – a Low noise treatment with ϵ_L, and a High noise treatment with $\epsilon_H > \epsilon_L$. Let Y_j^* denote the theory-implied strategic cutoff for treatment $j = \{L, H\}$. The theoretical predictions are summarized in Table 1.

We collected data on 8 groups in the High Noise treatment and 8 groups in the Low Noise treatment, a total of 160 subjects. The sessions were run in the BI Norwegian Business School Research Lab from May 2016 to June 2017. The experiment was programmed in z-Tree (Fischbacher, 2007) and subjects were recruited from the general student populations of BI Norwegian Business School and the University of Oslo using the software ORSEE (Greiner, 2015).

4 Results

Our first question is to what extent subjects follow the equilibrium requirement of using undominated cutoff strategies in our experiment. For each player i and each round t, let x_{it}^A be the highest signal at which subject i chooses A and x_{it}^B be the lowest signal at which she chooses B. We say that a subject's behavior is consistent with a cutoff strategy in round t, if $x_{it}^B \geq x_{it}^A$. Letting ϵ be the noise in each treatment ($\epsilon \in \{10, 20\}$), observe that playing B is dominated by A whenever $x_{it} < 20 - \epsilon$ and A is dominated by B whenever $x_{it} > 74 + \epsilon$. We say that a subject's behavior is consistent with an undominated cutoff strategy if it is consistent with a cutoff strategy, and $x_{it}^B \geq 20 - \epsilon$ and $x_{it}^A \leq 74 + \epsilon$.

Overall, the observed behavior of the subjects is largely consistent with playing undominated cutoff strategies. On average, 89 % of the subjects play in a way consistent with undominated cutoff strategies in the Low Noise treatment. In the High Noise treatment, the corresponding number is 92%. There is also some evidence of an increasing reliance on undominated cutoff strategies over time. Figure 1 shows the evolution in the use of cutoff strategies over time for each of our treatments. The percentage of subjects whose behavior is consistent with undominated cutoff strategies is increasing as play progresses.



Low Noise High Noise

Figure 1: Percentage of subjects, whose behavior is consistent with undominated cutoff strategies.

Result 1 (Cutoff strategies): *Subjects play consistently with undominated cutoff strate-*

gies.

To estimate strategic cutoffs we take the average, individual by individual, of the highest signal for which a subject chooses A and the lowest signal for which the subject chooses B. We then take the mean of these cutoffs within each group and refer to it as the Mean Estimated Threshold (MET).

In what follows we focus on first round behavior. The level-k theory explored in Section 5 below is meant to address initial play in unfamiliar environments, before learning kicks in (Crawford (1995)). Experimental evaluations using models of limited depths of reasoning, therefore, typically focus on first round behavior (Crawford, 1995, Camerer, 2011, chapters 1 and 6). Result 4 suggests that there is some learning going on in our data. Thus, focusing on first round behavior appears justified.[12]

Table 2 reports the mean estimated cutoff using first-round data and average behavior per group as observations. Data in the table are ranked in ascending order for each treatment based on the METs. As is evident, in each ordered pair of groups, the MET is higher in the High Noise treatment.

Group #	Low noise	High noise
1	22.8	31.8
2	30.2	35.8
3	35.9	39.4
4	38.0	41.3
5	39.9	44.6
6	40.8	48.7
7	44.4	49.5
8	44.8	51.5
Mean cutoff ($\bar{Y}_j$)	37.1	42.8
Standard deviation	7.4	6.9
Theoretical predictions	41.4	37.8

Table 2: Estimated strategic cutoff; first round data only, ranked groups.

[12]Note, however, that results are qualitatively and quantitatively similar when using all data and preforming a logit estimation of individuals likelihood of attacking conditional on their signals, treatment and an interaction-term.

Having obtained the METs across groups and treatments, we proceed to testing the model predictions.

A crucial comparative static of the model is that the MET in the High Noise treatment should be lower than in the Low Noise treatment. From Table 2 the observed difference, averaging over groups in each treatment, is the opposite: the MET in the High Noise treatment is 5.7 units higher than in the Low Noise treatment.

To formally test whether the difference across treatments is significant, we follow a conservative approach and run a Mann-Whitney U-test where we compare the rank-sums of METs using group averages as units of observation. The null hypothesis is that the MET is not higher in the Low Noise treatment than in the High Noise treatment. The results are shown in Table 3. We reject the null hypothesis with a p-value of 8%. Hence, the strategic cutoffs are *lower* in the Low Noise treatment compared to the High Noise treatment.

Treatment	Obs	Expected Rank sum	Rank sum
Low Noise	8	68	55
High Noise	8	68	81
	p-value		0.08*

Table 3: One-sided Mann-Whitney U-test of comparative statics.

Result 2 (Comparative statics): *The estimated strategic cutoffs are lower in the Low Noise treatment compared to the High Noise treatment.*

According to the model, the treatment difference between the estimated cutoffs should be approximately 3.6. As noted, the observed difference is -5.7. Table 4 reports the results from a two sided Mann-Whitney U-test where the null hypothesis is that the difference in METs is 3.6. Again, the test uses only first round data and has group averages as units of observation. We reject the null hypothesis that $\bar{Y}_L - \bar{Y}_H = 3.6$ with a p-value of 3%.[13]

[13]Note that $\bar{Y}_L - \bar{Y}_H = 3.6 \Rightarrow Y_L^* - \bar{Y}_L = Y_H^* - \bar{Y}_H$.

Treatment	Obs	Expected Rank sum	Rank sum
$Y_L^* - \overline{Y}_L$	8	68	48
$Y_H^* - \overline{Y}_H$	8	68	88
p-value			0.03**

Table 4: Two-sided Mann-Whitney U-test of treatment difference.

Result 3 (Treatment difference): *The treatment difference is significantly different from what is implied by theory.*

The evolution of the average estimated cutoffs over time is shown in Figure 2. There are some indications of learning, as cutoffs converge over time.

Result 4 (Evolution of play): *Some convergence of behavior over time.*

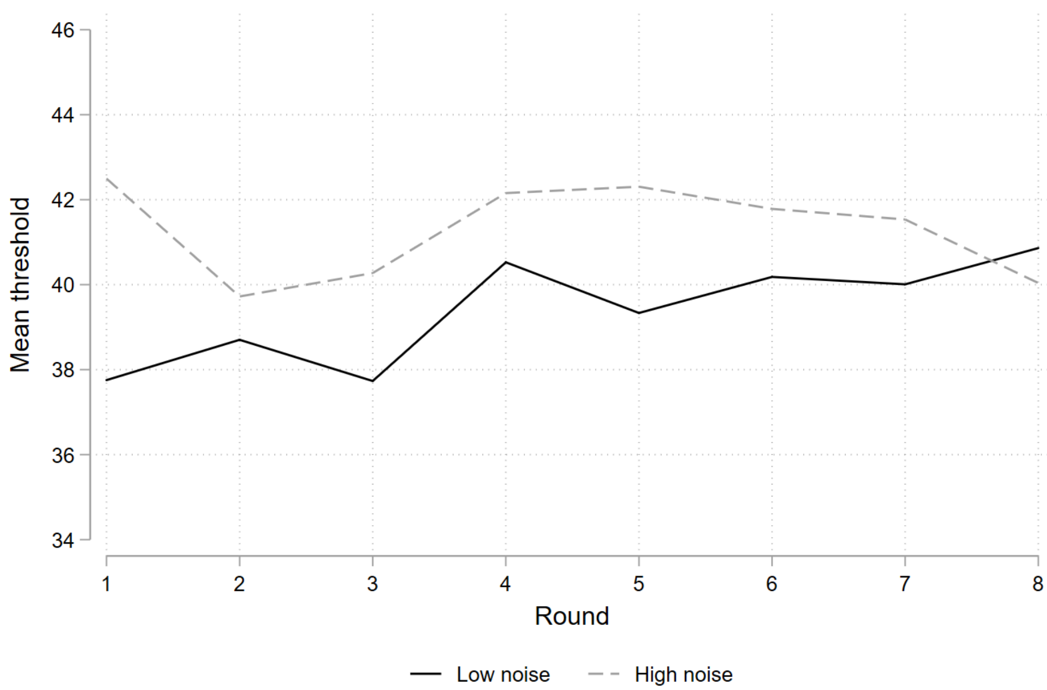


Figure 2: Average estimated strategic cutoffs for each treatment, by round.

5 Deviations from equilibrium theory

The results in the previous section suggest that the comparative statics of the strategic cutoff with respect to information quality go in the opposite direction relative to the theory. In this section, we propose one explanation for this finding.

5.1 Level-k thinking

Level-k thinking is a frequently used solution concept in Behavioral Game Theory.[14] It features limited depths of reasoning, adds a specific structure to agents' beliefs, and is particularly meant to capture players' initial behavior in strategic games, before learning induces higher levels of sophistication. In this framework, each player's type Lk is drawn from a discrete distribution over $k \in \{0, 1, \ldots, \infty\}$, where Lk denotes a type that engages in k rounds of reasoning. In particular, the behavior of $L0$ types is specified as a model primitive, and $L0$ types have zero mass. An $L1$ type best replies as if all other agents are $L0$ types, an $L2$ type assumes that all other agents are $L1$ types, and so on.

The main appeal of a level-k model in our setting is that it can change the comparative statics from the standard global games theory on how information dispersion affects players' actions and regime stability. In the standard global games model with fully rational types changes in information dispersion affect players' actions only through a "payoff sensitivity" effect (Iachan and Nenov (2015)). This effect is present when the net payoffs from attacking over not attacking, given regime-change or no regime-change, depend on the fundamental. In our specific environment, this effect implies that a higher value of information dispersion makes players more aggressive, since it increases their fundamental uncertainty and, consequently, their expected payoff conditional on regime change.

In the level-k model, unlike the fully rational model, a level-k player that best responds to level-$(k-1)$ players may end up playing according to a different cutoff strategy compared to

[14] See, for instance, Nagel (1995); Stahl and Wilson (1995); Kubler and Weizsacker (2004); Crawford, Costa-Gomes, and Iriberri (2013).

the level-$(k-1)$ types. This opens up the possibility for a novel effect of information quality on players' actions. To understand this effect, note that there is a strategic complementarity across types of different levels. Specifically, the aggressiveness of the level-$(k-1)$ types (i.e. the location of their cutoff) influences the level-k type's cutoff (and, through that cutoff, affects higher levels). Higher information dispersion *attenuates* this across-type strategic complementarity, since it makes players less coordinated when attacking and also reduces their ability to forecast the actions of other players. Therefore, if level-$(k-1)$ types are more aggressive than level-k types, higher information dispersion will make level-k types react *less* to the aggressiveness of level-$(k-1)$ types. Put differently, level-k types become *less* aggressive with higher information dispersion.

In the Appendix we formalize this "strategic attenuation" effect of higher information dispersion in the level-k model and also show that under some conditions it goes against and even dominates the "payoff sensitivity" effect. Specifically, the comparative statics with respect to information dispersion can be reversed, provided that there are sufficiently many agents that engage in few rounds of reasoning, and $L1$ types are relatively aggressive. In terms of model primitives, since $L1$ types tend to be more aggressive when they expect $L0$ types to play more aggressively, the comparative statics are reversed in a level-k model, provided that there are sufficiently many types that engage in few rounds of reasoning, and $L0$ types are expected to play sufficiently aggressively.

5.2 Empirical evaluation

In this section, we evaluate empirically whether the level-k model can account for the deviations from equilibrium theory that we have documented in Section 4.

5.2.1 Methodology

To separate the subjects into different level-k types, we follow Kneeland (2016) and estimate a finite mixture model on our experimental data.[15] We allow subjects to be L1, L2, and equilibrium types, which we denote by Lk, $k \in \{1, 2, \infty\}$.[16] We denote the share of Lk types by p_k, with p_∞ denoting the share of equilibrium types.

As highlighted in the previous section, a level-k model can lead to a reversal of the comparative statics, provided that two conditions are satisfied. First, there are sufficiently many types that engage in few rounds of reasoning and, second, $L0$ types are expected to play sufficiently aggressively. Therefore, in our empirical implementation, we assume that $L0$ types are expected to play aggressively. Specifically, we assume that $L1$ types believe that $L0$ types attack with probability 1.[17]

While most of the literature in which level-k models are used to explain data from experimental games assumes that $L0$ types randomize uniformly over actions (Crawford, Costa-Gomes, and Iriberri, 2013, and references therein), the literature on experimental coordination games has shown that initial play tends to be biased towards payoff dominant actions (Costa-Gomes, Crawford, and Iriberri, 2009).[18] Moreover, assuming that $L0$ types randomize uniformly in global games of regime change (so that the share of agents attacking is uniformly distributed) leads to the fully rational equilibrium, since $L1$ types end up holding (and reacting to) Laplacian beliefs about the remaining players' actions (Morris and Shin, 2003). For these reasons Kneeland (2016) assumes that $L0$ types play more aggressively than uniform randomization in her empirical investigation of a level-k model in an experimental global game. Our assumption of aggressive $L0$ types is, therefore, in line with this previous

[15] We follow the estimation procedure for finite mixture models in Mofatt (2016), chapter 8.

[16] As in Kneeland (2016), equilibrium types engage in infinite rounds of reasoning, so they play according to the equilibrium strategies from the global games model.

[17] In the Online Appendix we explore the robustness of our results with respect to this assumption when $L0$ types are expected to play less aggressively. Our empirical results also hold qualitatively for lower levels of aggressiveness by $L0$ types.

[18] A deviation from this assumption is also found in the literature on auctions, where L0 types are assumed to bid their value conditional on their own signal (Crawford and Iriberri, 2007).

work on level-k models in experimental global games. We also note that we do not treat the aggressiveness of $L0$ types as a free parameter that can vary across treatments.

Notice, however, that due to the across-type strategic complementarity in the level-k model, the assumption of aggressive $L0$ types reduces the strategic cutoffs for the behavioral types. Consequently, given the observed play in our experimental data, our estimation procedure would end up classifying the majority of players as equilibrium types. Therefore, to ensure that the *levels* of the strategic cutoffs of the Lk types are in line with our empirical data, we assume that players are risk-averse. Specifically we assume that players have constant relative risk aversion (CRRA) preferences and set the players' coefficient of relative risk aversion to 2/3, in line with estimates from the existing experimental literature (Harrison and Rutström, 2008). Introducing risk-aversion raises the strategic cutoffs for all types, while preserving the predictions of the level-k model with respect to changes in information dispersion.[19]

We further assume that each subject follows the action of a particular Lk type with some error. Specifically, in each decision round a subject makes a decision consistent with her type with probability $1 - \nu$ and makes an error with probability ν. If the player makes an error, the choice depends on an error density $d^k\left(a_q^i, \lambda\right)$ specified below.

Let $Q = \{1, 2, ..., 10\}$ denote the set of all decisions, $q \in Q$ denote a specific decision instance, and a_q^i denote the choice of subject i in instance q. For each subject×type, we define the set $Q^{ik} \subset Q$, which consists of all instances q, where subject i made a choice a_q^i consistent with type Lk. Weighting over the different types (k) and summing over all subjects (i), we get that the log-likelihood of observing a particular set of choices is

$$\mathcal{L} = \sum_{i=1}^{N} \log\left[\sum_{k=1}^{3} p_k \left(\Pi_{q\in Q^{ik}}\left(1 - \nu + \nu d^k\left(a_q^i, \lambda\right)\right)\right)\left(\Pi_{q\notin Q^{ik}} \nu d^k\left(a_q^i, \lambda\right)\right)\right]. \tag{7}$$

[19]Risk aversion on its own cannot reverse the comparative statics in a standard global game model, since it only dampens the "payoff sensitivity" effect. In the Appendix we present the estimation results with risk-neutral types. The estimated share of equilibrium types in that case is 71%.

Parameter	Estimate
Fraction of level-1 agents (p_1)	0.22
	[0.09]
Fraction of level-2 agents (p_2)	0.56
	[0.05]
Fraction of equilibrium types ($1 - p_1 - p_2$)	0.22
	[0.09]
Trembling rate (ν)	0.94
	[0.25]
Precision of error density (λ)	0.52
	[0.11]
n	1600

Table 5: Results from estimating equation (7) on data from round 1. Bootstrapped standard errors in brackets.

The parameter λ is a precision parameter in the error density

$$d^k\left(a_q^i, \lambda\right) = \frac{\exp\left\{\lambda S_q^k\left(a_q^i\right)\right\}}{\exp\left\{\lambda S_q^k\left(\text{attack}\right)\right\} + \exp\left\{\lambda S_q^k\left(\text{not attack}\right)\right\}}, \tag{8}$$

where $S_q^k\left(a_q^i\right)$ denotes the expected payoff of an agent of type Lk at decision instance q, who makes a choice a_q^i.

The unit of analysis is now individual decisions, which is in line with the existing literature on coordination experiments (Costa-Gomes, Crawford, and Iriberri, 2009; Crawford, Gneezy, and Rottenstreich, 2008). We fit 4 independent parameters, namely p_1, p_2 (the fractions of types $L1$ and L2), λ, and ν, on data from the first round for both treatments. In the numerical maximization of the likelihood function, we constrain all parameters to take on non-negative values.

5.2.2 Results

The results are shown in Table 5. The fraction of level-k types is estimated to be 78 %. This is roughly in line with the estimate from Kneeland (2016). Most agents are classified as L2, albeit with a relatively large probability of trembling.

We proceed by investigating whether the estimated level-k model can rationalize our experimental findings. A simple first pass is to compute the weighted average of the theory-implied strategic cutoffs, using the estimated distribution of Lk types from Table 5 as weights and investigating whether the resulting average cutoff in the Low Noise treatment is lower than the average cutoff in the High Noise treatment.[20] This yields an average strategic cutoff of 34.84 in the Low Noise treatment, and 40.17 in the High Noise treatment. Put differently, agents are, on average, *less* aggressive in the High Noise treatment compared to the Low Noise treatment, which is in line with our experimental findings.

A caveat with the preceding exercise is that it does not take into account that our estimated model allows for trembles. We proceed with the following simulation exercise: We simulate 1000 sessions, whereby in each session 640 games are played. For each session and game, agents draw a type according to the estimated type distribution from Table 5. We then assume that agents play according to their drawn type, with a probability of trembling equal to our estimated ν. Conditional on trembling, agents choose actions according to the spike-logit density ((8)), governed by the precision parameter λ. For each game, we then compare the mean cutoffs across a high and a low noise treatment, denoted by $\hat{\theta}_l$ and $\hat{\theta}_h$, respectively.

Figure 3 plots the empirical CDF of the differences between average strategic cutoffs across the two treatments ($\hat{\theta}_l - \hat{\theta}_h$) using the simulated data. More than 95% of the simulated sessions have negative differences in estimated strategic cutoffs, indicating that the comparative statics are flipped relative to equilibrium theory. We take this as evidence that the level-k model, combined with risk-averse players, can explain our experimental findings.

6 Concluding remarks

In this paper we experimentally test how changes in private information precision affect regime stability in a standard global games model. We show that contrary to the theoretical

[20]The theory-implied cutoffs in the Low Noise treatment are 20.56, 33.77 and 52.11 for L1, L2 and L∞ respectively. For the High Noise treatment, the corresponding cutoffs are 22.44, 42.68 and 51.69.

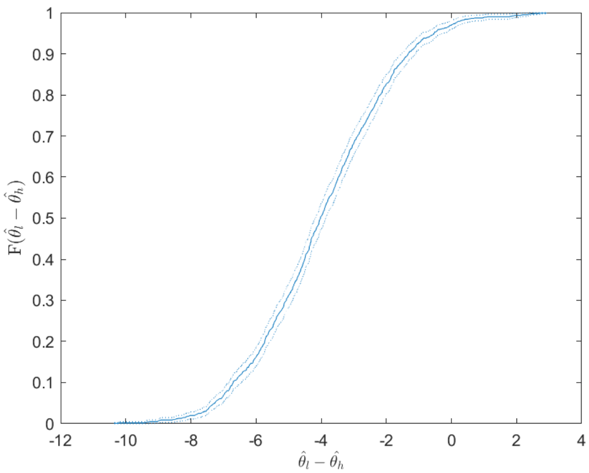


Figure 3: Empirical CDF of estimated cutoff differences from 1000 simulated sessions. Dotted lines indicate 95% confidence intervals as computed by Greenwoods formula.

predictions agents become less aggressive when information dispersion increases. We show that augmenting the standard global games set-up with boundedly rational agents that engage in level-k thinking can help explain our experimental finding. In the level-k model, information quality affects agents' actions through a novel channel, which does not operate in the fully rational model. Moreover, that novel channel can reverse the comparative statics with respect to changes in information precision.

The fact that the fully rational and level-k models can differ so dramatically in their predictions about the effect of information quality on behavior points to the importance of studying more carefully global coordination games with boundedly rational agents, both theoretically and experimentally. We view this as a promising venue for future research.

References

Angeletos, G.-M., and C. Lian (2017): "Dampening General Equilibrium: From Micro to Macro," Discussion paper, National Bureau of Economic Research.

Avoyan, A. (2017): "Communications in Global Games: Theory and Experiment," Discussion paper, Working Paper, Indiana University.

Cabrales, A., R. Nagel, and R. Armenter (2007): "Equilibrium selection through incomplete information: an experimental study," *Experimental Economics*, 10, 221–234.

Camerer, C. F. (2011): *Behavioral game theory: Experiments in strategic interaction.* Princeton University Press.

Carlsson, H., and E. van Damme (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989–1018.

Cooper, R., D. DeJong, R. Forsythe, and T. Ross (1990): "Selection criteria in coordination games: some experimental results," *American Economic Review*, 80(1), 218–233.

——— (1992): "Communication in coordination games," *Quarterly Journal of Economics*, 107(2), 739–771.

Cornand, C., and F. Heinemann (2014): "Measuring agents' reactions to private and public information in games with strategic complementarities," *Experimental Economics*, 17(1), 61–77.

Costa-Gomes, M. A., V. P. Crawford, and N. Iriberri (2009): "Comparing models of strategic thinking in Van Huyck, Battalio, and Beil's coordination games," *Journal of the European Economic Association*, 7(2-3), 365–376.

Crawford, V., M. Costa-Gomes, and N. Iriberri (2013): "Structural models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications," *Journal of Economic Literature*, 51(1), 5–62.

Crawford, V. P. (1995): "Adaptive dynamics in coordination games," *Econometrica: Journal of the Econometric Society*, pp. 103–143.

Crawford, V. P., U. Gneezy, and Y. Rottenstreich (2008): "The power of focal points is limited: even minute payoff asymmetry may yield large coordination failures," *The American Economic Review*, 98(4), 1443–1458.

Crawford, V. P., and N. Iriberri (2007): "Level-k auctions: Can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions?," *Econometrica*, 75(6), 1721–1770.

Edmond, C. (2013): "Information manipulation, coordination, and regime change," *Review of Economic Studies*, 80(4), 1422–1458.

Fischbacher, U. (2007): "z-Tree: Zurich toolbox for ready-made economic experiments," *Experimental Economics*, 10(2), 171–178.

Goldstein, I., and A. Pauzner (2005): "Demand-Deposit Contracts and the Probability of Bank Runs," *the Journal of Finance*, 60(3), 1293–1327.

Greiner, B. (2015): "Subject pool recruitement procedures: organizing experiments with orsee," *Journal of the Economic Science Association*, 1(1), 114–125.

Harrison, G. W., and E. Rutström (2008): "Risk aversion in the laboratory," in *Risk aversion in experiments*, pp. 41–196. Emerald Group Publishing Limited.

Heggedal, T.-R., L. Helland, and K.-E. N. Joslin (2018): "Should I Stay or Should I Go? Bandwagons in the Lab," *Journal of Economic Behavior and Organization*, 150, 86–97.

Heinemann, F., and G. Illing (2002): "Speculative attacks: unique equilibrium and transparency," *Journal of International Economics*, 58(2), 429–450.

Heinemann, F., R. Nagel, and P. Ockenfels (2004): "The theory of global games on test: Experimental analysis of coordination games with public and private information," *Econometrica*, 72(5), 1583–1599.

Heinemann, F., R. Nagel, and P. Ockenfels (2009): "Measuring Strategic Uncertainty in Coordination Games," *Review of Economic Studies*, 76, 181–221.

Iachan, F., and P. Nenov (2015): "Information quality and crises in regime-change games," *Journal of Economic Theory*, 158B, 739–768.

Kneeland, T. (2016): "Coordination under limited depth of reasoning," *Games and Economic Behavior*, 96, 49–64.

Kubler, D., and G. Weizsacker (2004): "Limited depth of reasoning and failure of cascade formation in the laboratory," *Review of Economic Studies*, 71(2), 425–441.

Mofatt, P. G. (2016): *Experimetrics. Econometrics for Experimental Economics.* Palgrave McMillan.

Morris, S., and H. Shin (2003): "Global Games: Theory and Applications," in *Advances in Economics and Econometrics, the Eighth World Congress*, ed. by H. Dewatripont, and Turnovsky.

Morris, S., and H. S. Shin (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *The American Economic Review*, 88(3), pp. 587–597.

Nagel, R. (1995): "Unraveling in guessing games: An experimental study," *American Economic Review*, 85(5), 1313–1326.

Rochet, J., and X. Vives (2004): "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?," *Journal of the European Economic Association*, 2(6), 1116–1147.

Stahl, D. O., and P. W. Wilson (1995): "On players models of other players: Theory and experimental evidence," *Games and Economic Behavior*, 10(1), 218–254.

Straub, P. (1995): "Risk dominance and coordination failures in static games," *Quarterly Review of Economics and Finance*, 35(4), 339–363.

Szkup, M., and I. Trevino (2017): "Sentiments, strategic uncertainty, and information structures in coordination games," *Mimeo*.

Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1990): "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure," *American Economic Review*, 80(1), 234–48.

Appendix

The effect of information quality with level-k thinking.

In this appendix, we provide an example of why level-k thinking is capable of generating the comparative statics that are in line with what we document in the experiment. Consider the set-up from Section 2, but assume that $N \to \infty$ for simplicity (i.e. we analyze a large game rather than a game with a finite number of players). Regime change occurs if a fraction $g(Y)$ of players attacks, where g is continuously differentiable and (weakly) decreasing in Y. Also, we consider $U(Y) = U < 0$ and define $d(Y) \equiv D(Y) - U$, as in our experimental game. Given the assumed properties of $D(Y)$, $d(Y)$ is also strictly increasing in Y. Assume that agents have limited depth of reasoning. Following Kneeland (2016), we assume that $L1$ agents believe that the aggregate behavior of $L0$ types is given by the cumulative distribution function $Q(z|Y)$, where z denotes the fraction of agents that attack. Here, $Q(z|Y)$ is continuously differentiable and weakly decreasing in Y, so that $L1$ types believe that a higher value of Y leads to a larger share of $L0$ types attacking.

L1 types

Consider an $L1$ type that observes a signal x_i and denote her net payoff from attacking vs. not attacking by

$$\begin{aligned} \pi_i^{L1} &= E_i[D(Y)|z > g(Y)] \Pr_i\{z > g(Y)\} + U \Pr_i\{z < g(Y)\} \\ &= E_i[d(Y)|z > g(Y)] \Pr_i\{z > g(Y)\} + U. \end{aligned}$$

Consider a typical $L1$ type that observes a signal $x_i \in [\epsilon, M - \epsilon]$, i.e. a signal which is not too close to the extremes of the support. Her posterior belief about Y is distributed uniformly on $[x_i - \epsilon, x_i + \epsilon]$ and we can write π_i^{L1} as,

$$\pi_i^{L1} = \int_{x_i-\epsilon}^{x_i+\epsilon} d\left(Y\right)\left[1 - Q\left(g\left(Y\right)|Y\right)\right] \frac{1}{2\epsilon} dY + U.$$

Define $\tilde{d}\left(Y\right) \equiv d\left(Y\right)\left[1 - Q\left(g\left(Y\right)|Y\right)\right]$. Therefore, $\tilde{d}$ combines the net payoff from attacking over not attacking, given a successful attack, $d\left(Y\right)$, with the probability of regime change occurring. Note that given the assumptions on d and Q, $\tilde{d}\left(Y\right)$ is strictly increasing in Y. Notice that $\frac{d\pi_i^{L1}}{dx_i} > 0$, so $L1$ types that observe a higher signal have a higher net payoff from attacking both because they expect a higher value of Y (i.e. weaker fundamental) but also because they believe that $L0$ agents will play more aggressively. Next, denote the strategic cutoff of $L1$ types by x_{L1}^*. That cutoff satisfies:

$$\hat{\pi}^{L1} := \int_{x_{L1}^*-\epsilon}^{x_{L1}^*+\epsilon} \tilde{d}\left(Y\right) \frac{1}{2\epsilon} dY + U = 0. \tag{9}$$

Agents with signals to the right (resp. left) of x_{L1}^* always (never) attack. Lemma 1 characterizes how the behavior of $L1$ types varies with signal precision.

Lemma 1. *In the game described above L1 agents attack according to monotone strategy with a cutoff x_{L1}^*, defined in (9). Furthermore,*

$$\frac{\partial x_{L1}^*}{\partial \epsilon} \propto -\int_{-1}^{1} \tilde{d}'\left(x_{L1}^* + \epsilon t\right) t \; dt.$$

Proof. We can use a change of variables $t = \frac{Y - x_{L1}^*}{\epsilon}$ to write $\hat{\pi}^{L1}$ as

$$\hat{\pi}^{L1} = \frac{1}{2}\int_{-1}^{1} \tilde{d}\left(x_{L1}^* + \epsilon t\right) dt + U.$$

Therefore,

$$\frac{\partial \hat{\pi}^{L1}}{\partial \epsilon} = \frac{1}{2}\int_{-1}^{1} \tilde{d}'\left(x_{L1}^* + \epsilon t\right) t dt$$

Since, $\frac{d\hat{\pi}^{L1}}{dx^*_{L1}} > 0$, by the implicit function theorem we have that,

$$\frac{\partial x^*_{L1}}{\partial \epsilon} \propto -\int_{-1}^{1} \tilde{d}'\left(x^*_{L1} + \epsilon t\right) t dt.$$

□

With the payoff structure as in our experiment, the behavior of $L1$ types will not change with changes in ε, since their perceived probability of regime change is noise invariant. Additionally, when noise is small, to a first-order approximation the effect of changes in noise on the behavior of $L1$ types is zero.

Example 1. Constant resilience and L0 types play independently of Y: If $d(Y) = Y$, $g(Y) = g$ and $Q(z|Y) = Q(z)$,$-\int_{-1}^{1} \tilde{d}'\left(x^*_{L1} + \epsilon t\right) t dt = 0$.

Example 2. Small noise: Noise independence holds in the limit, as $\lim_{\epsilon\to 0} \int_{-1}^{1} \tilde{d}'\left(x^*_{L1} + \epsilon t\right) t \; dt = \tilde{d}'\left(\lim_{\epsilon\to 0} x^*_{L1}\right) \int_{-1}^{1} t \; dt = 0$.

L2 Types

Next, consider the $L2$ types. Define Y^f_{L1} as

$$g\left(Y^f_{L1}\right) = \frac{Y^f_{L1} + \epsilon - x^*_{L1}}{2\epsilon} = \frac{1}{2} + \frac{Y^f_{L1} - x^*_{L1}}{2\epsilon}. \tag{10}$$

Therefore, $L2$ types believe that regime change takes place if $Y > Y^f_{L1}$.[21]

As with the $L1$ types, consider a $L2$ type that observes a signal x_i and denote her net payoff from attacking vs. not attacking by

$$\begin{aligned}\pi^{L2}_i &= \Pr_i\left\{Y > Y^f_{L1}\right\} E_i\left[D(Y)\,|Y > Y^f_{L1}\right] + U \Pr_i\left\{Y < Y^f_{L1}\right\} \\ &= \Pr_i\left\{Y > Y^f_{L1}\right\} E_i\left[d(Y)\,|Y > Y^f_{L1}\right] + U.\end{aligned}$$

[21] We suppose further that $Y^f_{L1} \in [2\epsilon, M - 2\epsilon]$, to abstract away from complications arising because of the bounded support of the uniformly distributed private signals.

$L2$ types that observe a signal $x_i \in \left[Y^f_{L1} - \epsilon, Y^f_{L1} + \epsilon\right]$ are not sure of either regime survival or failure, so this is the signal interval of interest for the strategic cutoff characterization. They have a posterior belief about Y, which is distributed uniformly on $[x_i - \epsilon, x_i + \epsilon]$, and we can write π_i^{L2} as

$$\pi_i^{L2} = \int_{Y^f_{L1}}^{x_i+\epsilon} d(Y) \frac{1}{2\epsilon} dY + U.$$

As with $L1$ types, $\frac{d\pi_i^{L2}}{dx_i} > 0$, so $L2$ types that observe a higher signal also have a higher net payoff from attacking. We denote the strategic cutoff of $L2$ types by x^*_{L2}, which satisfies

$$\hat{\pi}^{L2} := \int_{Y^f_{L1}}^{x^*_{L2}+\epsilon} d(Y) \frac{1}{2\epsilon} dY + U = 0. \tag{11}$$

Lk Types

One can proceed recursively and define the strategic cutoff for an Lk type, $k \geq 2$, x^*_{Lk} as the solution to

$$\int_{Y^f_{L(k-1)}}^{x^*_{Lk}+\epsilon} d(Y) \frac{1}{2\epsilon} dY + U = 0, \tag{12}$$

where $Y^f_{L(k-1)}$ is the failure cutoff given optimal behavior by $L(k-1)$ types, defined as

$$g\left(Y^f_{L(k-1)}\right) = \frac{1}{2} + \frac{Y^f_{L(k-1)} - x^*_{L(k-1)}}{2\epsilon}. \tag{13}$$

Strategic attenuation effect of higher information dispersion

We next show that changes in information quality ϵ may change the optimal behavior of Lk types differently from the effect of information quality on equilibrium play. Specifically, we show two results to that effect. The first is for an arbitrary distribution of level-k types but a more specific payoff function d. The second is for a model with just $L1$ and $L2$ types but with a more general payoff function d.

First, we assume that $d(Y) = \overline{d}$. We make this assumption to completely switch off the “payoff sensitivity” effect from changes in information dispersion ϵ, emphasized in Iachan and Nenov (2015), when d is a function of Y. Specifically, this effect implies that a higher value of ϵ makes agents more aggressive, since it increases fundamental uncertainty and, consequently, the expected payoff conditional on regime change. This ”payoff sensitivity” effect is the only effect that operates in a global games model when agents engage in equilibrium play. Switching off this effect implies no effect of information precision on equilibrium play. For simplicity, we also assume that we have a linear resilience function $g(Y) = \alpha + \beta Y$.

Proposition 1. *Consider the game described above and let $d(Y) = \overline{d}$, and $g(Y) = \alpha + \beta Y$, $\beta < 0$. Let x^*_{Lk} be the strategic cutoff of a Lk type, for $k \geq 2$, defined as in ((12)) with the cutoff for L1 types as defined in (9).*

- *If $x^*_{Lk} > x^*_{Lk-1}$, then $\frac{\partial x^*_{Lk}}{\partial \epsilon} < 0$, so, higher noise makes Lk types more aggressive.*
- *If $x^*_{Lk} < x^*_{Lk-1}$, then $\frac{\partial x^*_{Lk}}{\partial \epsilon} > 0$, so, higher noise makes Lk types less aggressive.*
- *If $x^*_{Lk} = x^*_{Lk-1}$, then $\frac{\partial x^*_{Lk}}{\partial \epsilon} = 0$, so, higher noise has no effect on the behavior of Lk types.*

Proof. Define $h(x)$ implicitly by

$$\int_{Y^f(x)}^{h(x)+\epsilon} \overline{d}\frac{1}{2\epsilon} dY + U = 0, \tag{14}$$

where $Y^f(x)$ is the failure cutoff if agents play according to a cutoff x, i.e.

$$g\left(Y^f(x)\right) = \frac{1}{2} + \frac{Y^f(x) - x}{2\epsilon}. \tag{15}$$

Therefore, $h(x)$ is the cutoff for an agent that believes everybody else plays according to cutoff x. Notice then that $x^*_{Lk} = h\left(x^*_{L(k-1)}\right)$, for $k \geq 2$. Also, if x^*_∞ denotes equilibrium play, x^*_∞ solves $x^*_\infty = h(x^*_\infty)$. Therefore, $x^*_{Lk} \to x^*_\infty$. Next, note that

$$h'(x) = \frac{1}{1 - 2\epsilon g'\left(Y^f(x)\right)} = \frac{1}{1 - 2\epsilon\beta} < 1.$$

Therefore, $\{x^*_{Lk}\}_{k=1}^{\infty}$ defines a monotone sequence, so that if $x^*_{L1} < x^*_{\infty}$, then $x^*_{L(k-1)} < x^*_{Lk} < x^*_{\infty}$ and vice versa for $x^*_{L1} > x^*_{\infty}$. Noting that $\frac{\partial}{\partial \varepsilon}(h'(x)) < 0$, it follows that an increase in ε leads to flattening of $h(x)$. Furthermore, since $\frac{\partial x^*_{\infty}}{\partial \varepsilon} = \frac{\partial h(x^*_{\infty})}{\partial \epsilon} = 0$, it follows that for $x < x^*_{\infty}$, $h(x)$ must strictly increase, while for $x > x^*_{\infty}$, $h(x)$ must strictly decrease. Noting that $x^*_{Lk} > x^*_{Lk-1}$, $\forall k \geq 2$ implies $x^*_{Lk} < x^*_{\infty}$ and vice versa for $x^*_{Lk} < x^*_{Lk-1}$, we arrive at the first and second observation in the proposition. Finally, since $x = h(x)$ has a unique solution (by uniqueness of the global games equilibrium), it follows that $x^*_{Lk} = x^*_{L(k-1)}$ implies $x^*_{Lk} = x^*_{\infty}$, $\forall k \geq 1$, which leads to the third observation. □

Intuitively, an increase in the dispersion of private noise attenuates the strategic complementarity across level-k types, since it makes agents less coordinated when attacking and also reduces their ability to forecast the actions of other agents. At the equilibrium cutoff that effect is not relevant, however, if agents best respond to agents that do not play according to the equilibrium cutoff, as in the level-k model, that effect becomes important.

Next, we show that this "strategic attenuation" effect operates also when the payoff function d depends on Y, so that the "payoff sensitivity" effect is not switched off. Suppose, for illustration, that there are only two level-k types: $L1$ and $L2$. We next show that the behavior of $L2$ types with respect to changes in the signal precision depends on a comparison of the strategic cutoffs of $L1$ and $L2$ players similar to the comparison from Proposition (2).

Proposition 2. *Consider the game described above and let $g(Y) = g$ and $Q(z|Y) = Q(z)$. Let x^*_{L1} and x^*_{L2} be the strategic cutoff of L1 and L2 types as defined in (9) and (11) above.*

- *If $x^*_{L1} > x^*_{L2}$, then $\frac{\partial x^*_{L2}}{\partial \epsilon} < 0$, so, higher noise makes L2 types more aggressive.*
- *There exists a $\Delta > 0$, such that for $x^*_{L1} < x^*_{L2} - \Delta$, $\frac{\partial x^*_{L2}}{\partial \epsilon} > 0$, so, higher noise makes L2 types less aggressive.*

Proof. From Lemma 1, x^*_{L1} does not depend on ϵ given $g(Y) = g$ and $Q(z|Y) = Q(z)$. We use a change of variables $t = \frac{Y - x^*_{L2}}{2\epsilon}$ to write $\hat{\pi}^{L2}$ as

$$\hat{\pi}^{L2} = \int_{\frac{Y^f_{L1} - x^*_{L2}}{2\epsilon}}^{1/2} d\left(x^*_{L2} + 2\epsilon t\right) dt + U.$$

Therefore,

$$\frac{\partial \hat{\pi}^{L2}}{\partial \epsilon} = -\left(-\frac{Y^f_{L1} - x^*_{L2}}{2\epsilon^2} + \frac{1}{2\epsilon}\frac{\partial Y^f_{L1}}{\partial \epsilon}\right) d\left(Y^f_{L1}\right) + \int_{\frac{Y^f_{L1} - x^*_{L2}}{2\epsilon}}^{1/2} d'\left(x^*_{L1} + 2\epsilon t\right) 2t dt.$$

Notice that

$$\frac{\partial Y^f_{L1}}{\partial \epsilon} = \frac{1}{\epsilon}\frac{Y^f_{L1} - x^*_{L1}}{1 - 2\epsilon g'\left(Y^f_{L1}\right)}.$$

After undoing the change of variables, we have,

$$\frac{\partial \hat{\pi}^{L2}}{\partial \epsilon} = -\left(-\frac{Y^f_{L1} - x^*_{L2}}{2\epsilon^2} + \frac{1}{2\epsilon^2}\frac{Y^f_{L1} - x^*_{L1}}{1 - 2\epsilon g'\left(Y^f_{L1}\right)}\right) d\left(Y^f_{L1}\right) + \frac{1}{\epsilon}\int_{Y^f_{L1}}^{x^*_{L2}+\epsilon} d'\left(Y\right)\left(Y - x^*_{L2}\right)\frac{1}{2\epsilon} dY.$$

Therefore, by the Implicit Function theorem,

$$\frac{\partial x^*_{L2}}{\partial \epsilon} \propto \left(x^*_{L2} - Y^f_{L1} + \frac{Y^f_{L1} - x^*_{L1}}{1 - 2\epsilon g'\left(Y^f_{L1}\right)}\right) d\left(Y^f_{L1}\right) - \int_{Y^f_{L1}}^{x^*_{L2}+\epsilon} d'\left(Y\right)\left(Y - x^*_{L2}\right) dY.$$

With $g\left(Y\right) = g$, $g' = 0$, so

$$\frac{\partial x^*_{L2}}{\partial \epsilon} \propto \left(x^*_{L2} - x^*_{L1}\right) d\left(Y^f_{L1}\right) - \int_{Y^f_{L1}}^{x^*_{L2}+\epsilon} d'\left(Y\right)\left(Y - x^*_{L2}\right) dY.$$

Furthermore, with $d\left(Y\right) = Y$, $d'\left(Y\right) = 1$, so,

$$\begin{aligned}\frac{\partial x^*_{L2}}{\partial \epsilon} &\propto \left(x^*_{L2} - x^*_{L1}\right) Y^f_{L1} - \left(\frac{1}{2}Y^2 - x^*_{L2}Y\Big|_{Y^f_{L1}}^{x^*_{L2}+\epsilon}\right) \\ &\propto \left(x^*_{L2} - x^*_{L1}\right) Y^f_{L1} + \frac{1}{2}\left(Y^f_{L1} - x^*_{L2}\right)^2 - \frac{1}{2}\epsilon^2.\end{aligned}$$

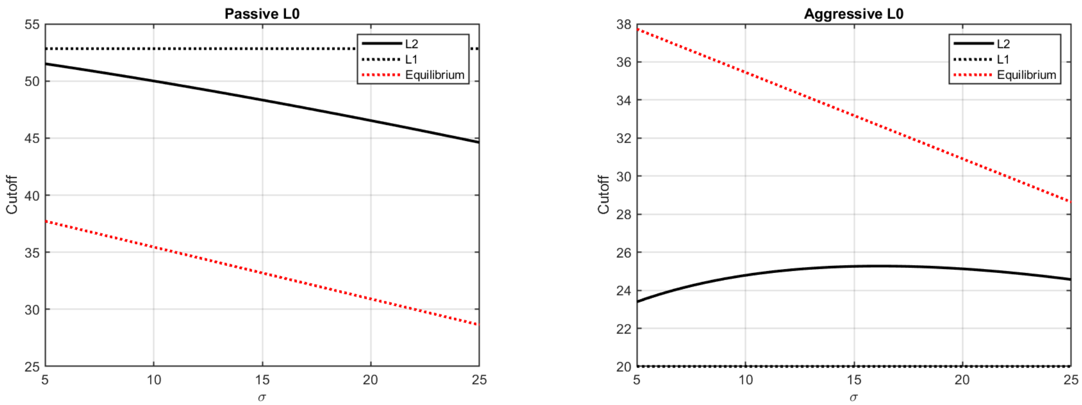


Figure 4: Illustrative example of Proposition 5. In the left panel, L0 agents attack with probability 0.45. In the right panel, L0 agents attack with probability 1. We assume $g = 6.5$ and that payoffs are in line with our experimental implementation, i.e. $D(Y) = Y - 20$ and $U = -20$.

Note that

$$\left(x_{L2}^{*} - x_{L1}^{*}\right) Y_{L1}^{f} + \frac{1}{2}\left(Y_{L1}^{f} - x_{L2}^{*}\right)^{2} - \frac{1}{2}\epsilon^{2} \geq \left(x_{L2}^{*} - x_{L1}^{*}\right) Y_{L1}^{f} - \frac{1}{2}\epsilon^{2} \geq \left(x_{L2}^{*} - x_{L1}^{*}\right) 2\epsilon - \frac{1}{2}\epsilon^{2},$$

since $Y_{L1}^{f} \geq 2\epsilon$. Then the sign of $\frac{\partial x_{L2}^{*}}{\partial \epsilon}$ depends on a comparison of x_{L2}^{*} and x_{L1}^{*}. Suppose that $x_{L2}^{*} \leq x_{L1}^{*}$. Then $\frac{\partial x_{L2}^{*}}{\partial \epsilon} < 0$. Suppose that $x_{L2}^{*} - x_{L1}^{*} > \frac{\epsilon}{4}$. Then, $\frac{\partial x_{L2}^{*}}{\partial \epsilon} > 0$. Setting $\Delta = \frac{\epsilon}{4}$, we arrive at our result. □

Figure 4 shows two examples based on the payoff parametrization from our experiment that confirm that both cases of Proposition 2 are relevant. In the left panel, L1-types believe that L0-types play relatively cautiously, so they also play relatively cautiously compared to the equilibrium (fully rational) types. In the right panel, L1-types believe that L0-types play relatively aggressively, so they tend to play more aggressively relative to the equilibrium types.

The intuition for why the effect of information precision is ambiguous follows from a comparison of the "payoff sensitivity" and "strategic attenuation" effects. As in the case of fully rational agents, there is a "payoff sensitivity" effect when d depends on Y. When $x_{L1}^{*} > x_{L2}^{*}$, the "strategic attenuation" effect reinforces the "payoff sensitivity" effect. However, when $x^*_{L1} < x^*_{L2}$, the "strategic attenuation" and "payoff sensitivity" effects oppose each other. When x^*_{L1} is sufficiently different from x^*_{L2}, the "strategic attenuation" effect ends up dominating the "payoff sensitivity" effect and the comparative statics reverse – $L2$ types become *less* aggressive.

As agents engage in more rounds of reasoning (and approach the fully rational types) the difference in the strategic cutoffs between level-k and level-$(k-1)$ types tends to decrease (as they converge to the strategic cutoff of the fully rational types). Consequently, the "strategic attenuation" effect tends to weaken. Therefore, the comparative statics with respect to information dispersion can be reversed, provided that there are sufficiently many agents that engage in few rounds of reasoning and $L1$ types are relatively aggressive. In terms of model primitives, since $L1$ types tend to be more aggressive when they expect $L0$ types to play more aggressively, the comparative statics are reversed in a level-k model, provided that there are sufficiently many types that engage in few rounds of reasoning and $L0$ types are expected to play sufficiently aggressively.

Parameter	Estimate
Fraction of level-1 agents (p_1)	0.07
	[0.04]
Fraction of level-2 agents (p_2)	0.22
	[0.08]
Fraction of equilibrium types ($1 - p_1 - p_2$)	0.71
	[0.07]
Trembling rate (ν)	0.42
	[0.09]
Precision of error density (λ)	0.02
	[0.01]
n	1600

Table 6: Results from estimating equation (7) on data from round 1. Bootstrapped standard errors in brackets.

Additional empirical results

Level-k model estimation with risk-neutral agents

The results from estimating the level-k model with risk-neutral agents are shown in Table 6.

Using the estimated fractions and the strategic cutoffs from the different types yields a average cutoff of 34.95 in the High noise treatment and a average cutoff of 37.14 in the Low Noise treatment.

Level-k model estimation with less aggressive L0

In our baseline specification, we assume that L0 types attack with probability 1. As a robustness exercise, we have redone the estimation assuming that L0 types are expected attack with probability 0.8. The results are shown in Table 7.

Using the estimated fractions and the strategic cutoffs from the different types yields a average cutoff of 44.2 in the High noise treatment and a average cutoff of 41.36 in the Low Noise treatment.

Parameter	Estimate
Fraction of level-1 agents (p_1)	0.82
	[0.10]
Fraction of level-2 agents (p_2)	0.00
	[0.05]
Fraction of equilibrium types ($1 - p_1 - p_2$)	0.18
	[0.09]
Trembling rate (ν)	0.60
	[0.16]
Precision of error density (λ)	0.27
	[0.09]
n	1600

Table 7: Results from estimating equation (7) on data from round 1. Bootstrapped standard errors in brackets.