Information Quality and Crises in Regime-Change Games

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Abstract

When crises potentially originate from coordination failures, does a deterioration in the quality of the information available to market participants contribute towards instability? We address this question in a general global game of regime change with a unique equilibrium and illustrate the implications in a debt rollover application. We show that a reduction in the quality of information increases the likelihood of regime change, thus reducing stability, when the net payoff in the case of a successful attack is more sensitive to the fundamentals than the net payoff in the case of status quo survival. We also discuss welfare implications.

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1 Introduction

Information quality is a central concern in macroeconomics and finance. During major disruptive events, such as debt runs or currency crises, market participants have to rely on imprecise information to make their decisions. Additionally, these decisions often include important coordination aspects. However, what is the effect of a change in information quality on financial stability?

We examine this question in a general global game of regime change. This environment features imperfectly informed agents facing a binary decision of whether to attack the status quo. A strategic complementarity arises from discrete payoff changes that occur when a sufficiently large group of agents chooses to attack and trigger regime change. An imperfectly observed state variable (the fundamentals) determines the strength of the status quo in terms of the minimum fraction of agents that must attack in order for regime change to occur. Importantly, the state also directly affects agent payoffs in the events of both regime change and status quo survival. Specifically, the net payoff from attacking the status quo relative to not attacking decreases weakly in the state, as stronger fundamentals weaken the incentives to attack. Indeed, the sensitivity of payoffs to the fundamentals is a central driver of the results we present.

The unique equilibrium of this game is characterized by two endogenous thresholds. The first is a strategic threshold, which determines the value of the private signal realization below which an agent attacks the status quo. The second is a regime-change threshold, which determines the value of the fundamentals below which regime change occurs.

Our main exercise consists of studying under which conditions a decrease in the precision of private information increases the regime-change threshold and thus makes the status quo less stable. This occurs whenever the sensitivity of the net payoff to the fundamentals is higher in the case of regime change than in the case of status quo survival. It reduces the regime change threshold in the opposite case, when the net payoff given a regime change is less sensitive than the net payoff given the survival of the status quo.

The intuition for this result is the following. Consider the game under perfect information. If an agent is sure of regime change, she finds it optimal to attack. In other words, the net payoff in the case of regime change is positive. Analogously, if an agent is sure of regime survival, the optimal action is to refrain from attacking: the net payoff given survival is negative. In the imperfect-information game, the probability-weighted net payoff associated with regime change provides incentives to attack, while the weighted net payoff in the case of survival provides opposing incentives. Less precise information makes more extreme realizations of the fundamentals more likely, increasing the absolute value of both weighted net payoffs. The increase in the value of the first one is larger than the change in the value of the second one whenever the net payoff given regime change is more sensitive. Therefore, more agents favor attacking, and the regime change threshold increases.

The model nests two important special cases that further clarify this mechanism: currency crises and bank runs. As commonly modeled, these cases allow only one of the two payoff differentials to be sensitive to the fundamentals. In the case of currency crises (as in Morris and Shin, 1998), payoffs are sensitive to the fundamentals only in the case that the currency peg is abandoned (regime change). In contrast, a bank run model based on Goldstein and Pauzner (2005) features sensitive payoffs only when the bank survives (status quo survival). As a consequence, a decrease in information quality has a destabilizing effect in the former and a stabilizing effect in the latter. Moving away from these cases, we present a debt rollover environment in which both payoff differentials respond to fundamentals.

We initially focus on the case in which information is purely private and priors are improper. We then extend our results to the case in which agents have informative priors regarding the fundamentals or receive public signals that create some degree of common knowledge. We study the effects of a change in *absolute* information quality (the overall precision of an agent's posterior beliefs) and the effects of a change in *relative* information quality (the relative precision of public and private information). We show that relative payoff sensitivities matter for the effects of changes in absolute information quality. In contrast, the effects of changes in relative information quality depend solely on the position of the prior mean.

We additionally discuss welfare in this framework. An externality is present because agents fail to internalize the effect of their individual actions on the determination of the equilibrium regime. This creates scope for welfare-improving policy, particularly through policies that affect information quality. We also extend the model to allow for externalities from agents' actions in the net payoff differentials of other agents in addition to the determination of the regime-change cutoff. This case is of particular interest for bank run modeling, as payoffs upon liquidation typically depend on the fraction of agents that choose to run on the bank. We show that our main result continues to hold in that case. Finally, we study other dimensions of information quality besides the precision of information and link our results to the growing literature on endogenous information acquisition.

Global games of regime change have been widely applied in the modeling of crisis episodes, such as currency attacks (Morris and Shin, 1998, Corsetti, Dasgupta, Morris, and Shin, 2004, and Guimaraes and Morris, 2007), debt crises (Corsetti, Guimaraes, and Roubini, 2006), bank runs (Rochet and Vives, 2004 and Goldstein and Pauzner, 2005) and short-term debt rollover problems (Morris and Shin, 2004).[1] Many of these contributions apply the insights of Carlsson and van Damme (1993) to derive a unique equilibrium in the presence of imperfect information in an environment otherwise characterized by multiplicity.

Despite its significance, information quality has, for the most part, not been the focus of these studies, which have instead focused on other aspects of the information structure of agents or on the nature of equilibria.[2] In our setting, the noisy information structure is more than a device used

[1]Beyond crisis modeling, regime-change games have been extensively used in other contexts, for example, to understand issues in investment (Dasgupta, 2007) and political change (Edmond, 2013), among others. Regime-change games have also been used in dynamic contexts (Angeletos, Hellwig, and Pavan, 2007) and particularly for modeling dynamic debt runs (He and Xiong, 2012a).

[2]Notable exceptions are Angeletos, Hellwig, and Pavan (2006) and Angeletos, Hellwig, and Pavan (2007), who also study the implications of different information structures on regime stability in global games of regime change. In the

to derive a unique equilibrium prediction, but rather the central element of the analysis. The rich payoff structure we allow for, which nests most of this literature, is central to the description of the equilibrium consequences of a change in information quality.

Recent papers have described how changes in information quality affect the probability of regime change in particular environments that feature a one-sided payoff sensitivity. Angeletos and Pavan (2013) examine a regime change game with payoff sensitivity in the case of regime change that generalizes the currency attack model of Morris and Shin (1998). They show that a decrease in the precision of private information increases the regime-change cutoff, making crises more frequent, a result that can be understood using our comparison of relative payoff sensitivities.[3] In the other direction, Moreno and Takalo (2011) investigate the effect of transparency in a bank-run model and find that in their framework increasing precision actually increases the probability of a bank run. The conditions we study in our paper shed light on this result as well.

This paper's emphasis on the effect of information quality on equilibrium behavior in a coordination game relates the paper to the work by Morris and Shin (2002), Angeletos and Pavan (2004), and Angeletos and Pavan (2007), who consider the welfare effects of more precise public and private information in economies with strategic complementarities. In particular, we use insights from the latter two papers and decompose the effects of absolute and relative information quality in the presence of an informative prior.

The focus of our paper on the information structure itself is shared by a growing literature on endogenous information acquisition (Hellwig and Veldkamp, 2009, Myatt and Wallace, 2012, Colombo, Femminis, and Pavan, 2013, Yang, 2013, Szkup and Trevino, 2013). Our analysis complements this literature by examining how particular changes in the information structure (i.e. information quality) map into changes in equilibrium regime determination.

Finally, our motivating environment relates to papers that study the effects of information in bank run models without elements of dispersed information (Chen, 1999, and Chen and Hasan, 2006). In addition, the environment is related to recent work that addresses issues of transparency, complexity, and information quality in the context of financial crises (Dang, Gorton, and Holmstrom, 2009, Caballero and Simsek, 2013, Vives (2014)) as well as the work on the importance of rollover risk to financial stability (Acharya, Gale, and Yorulmazer, 2011, He and Xiong, 2012b). Our paper complements this literature by studying how changes in the quality of individual information can affect financial stability by influencing the coordination between dispersed short-term creditors.

The remainder of the paper is organized as follows. Section 2 presents the general model of a regime-change game with flexible payoffs, provides applications, defines the equilibrium concept, and derives a uniqueness result. Section 3 presents the main results of the paper: the relative sensitivity condition and how it determines the effect of information quality on the regime-change

first paper, information quality is endogenous due to the signaling effects of policy interventions. In the second paper, agents accumulate signals over time and observe whether the regime has survived past attacks.

[3] Heinemann and Illing (2002) report a result in a similar spirit for a particular example of the currency attack model.

cutoff. Section 4 provides several extensions, including an analysis of information quality in the presence of public information and a welfare discussion. The last section concludes the paper.

2 Model

We describe a general set-up that focuses on the coordination problem among atomistic agents engaged in a regime-change game. Several examples of specific environments that are nested in our framework are described afterwards, including our main motivating example of a debt rollover crisis.

2.1 General environment

The economy is characterized by a continuum unit measure of agents indexed by $i \in [0,1]$ and a state $\theta \in \mathbb{R}$, which nature draws from a distribution to be described shortly. Depending on the specific application, θ may reflect the fundamentals of the economy or of the financial institution which determine the agents' final payoffs. Throughout the analysis, a higher θ is associated with stronger fundamentals. Agents simultaneously choose to take a binary action $a_i \in \{0,1\}$. A denotes the measure of agents choosing $a_i = 1$, so that $A = \int_i a_i di$ is the average action in the economy.

As a result, one of two regimes $\mathcal{R} \in \{0,1\}$ is realized depending on A and a critical threshold $g(\theta)$. Regime change ($\mathcal{R} = 1$) occurs whenever $A > g(\theta)$, while the status quo ($\mathcal{R} = 0$) is preserved whenever $A \leq g(\theta)$. The function $g(\theta)$ is assumed to be continuously differentiable and (weakly) increasing in θ: a higher value of θ implies that a (weakly) higher value of A is necessary for regime change to occur.

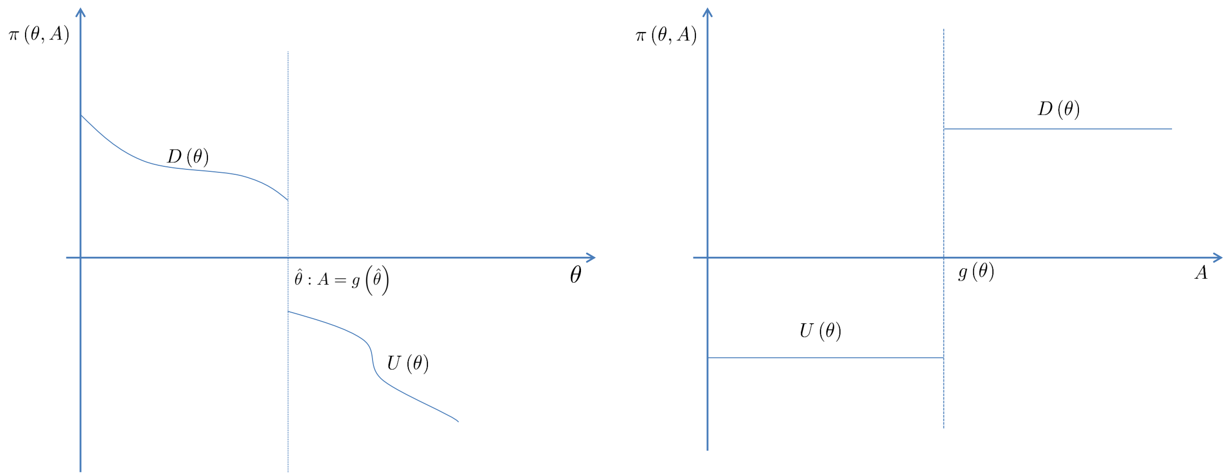
We have two main applications in mind, which are detailed in Section 2.2. In both currency attacks and debt rollover games, we associate action $a_i = 1$ with an action that induces the collapse of the currency peg or of the borrower. Analogously, regime change is related to this failure episode. A higher value of θ in both environments determines the capacity to survive a larger attack by speculators or a run by lenders.

Given the binary action structure, agents only take into account the payoff differential from the two alternatives when choosing their optimal action. To streamline the presentation and bring more clarity to our theoretical results, we describe the model in terms of these payoff differentials directly. In Section 2.2, we provide examples that illustrate how such net payoff differentials may be derived in important applications. In particular, the net payoff from choosing $a_i = 1$ relative to $a_i = 0$ is:

$$\pi(\theta, A) \equiv \begin{cases} U(\theta) & , if\ A \leq g(\theta) \\ D(\theta) & , if\ A > g(\theta) \end{cases}, \tag{1}$$

in which $U(\theta) < 0$ is the net payoff in the case of status quo survival and $D(\theta) > 0$ is the net payoff in the case that regime change occurs. We assume that $U(\theta)$ and $D(\theta)$ are bounded and non-increasing in θ. That is, a higher value of the state θ always biases agents' choice towards

$a_i = 0$.[4] Figure 1 illustrates the function $\pi(\theta, A)$ if one holds A constant while varying θ on the left-hand side panel or holds θ constant while varying A in the second panel.[5]



(a) Fixed A.

(b) Fixed θ.

Figure 1: Net payoff from $a_i = 1$.

Agents have imperfect information about the state θ. In particular, all agents have a common prior over θ given by $\theta = \theta_0 + \sigma_\theta \epsilon$ with $\theta_0 \in \mathbb{R}$ and $\sigma_\theta > 0$, and the random noise ϵ is distributed according to the cumulative distribution function F_ϵ with zero mean. As in the rest of the global games literature, one can think of the common prior as resulting from the incorporation of public information, the realization of which is common knowledge across agents. In that interpretation, σ_θ represents the quality of public information.

Additionally, each agent i observes a private signal $\theta_i = \theta + \sigma_\eta \eta_i$ with $\sigma_\eta > 0$, in which η_i is a mean-zero random variable that is independently and identically distributed according to the cumulative distribution function F_η. Furthermore, η_i is independent of ϵ and θ for all i. As standard, we define an expectation operator $E_{\theta_i}[f(\theta)]$ as the expectation with respect to the posterior distribution of an agent that has received a signal θ_i, and $\Pr_{\theta_i}(\theta \in Z) \equiv E_{\theta_i}[\mathbf{1}_Z]$ as the posterior probability distribution. We assume that F_ϵ and F_η have full support on the real line and admit densities f_ϵ and f_η, respectively.[6]

We will start our theoretical analysis by focusing on the limiting case of an uninformative public signal (a uniform prior over the entire real line) but return to the informative prior case in Section 4.1 for the case where ϵ and η_i are normally distributed. The main advantage of this initial approach

[4]Note that for equilibrium uniqueness, we will require at least one of $U(\theta)$, $D(\theta)$ and $g(\theta)$ to be strictly monotone in θ.

[5]We generalize these payoffs by allowing an impact of A on both differentials in Section 4.2 and show that our main result still holds.

[6]Our main result extends to the case where signals are uniformly distributed and the private signal is sufficiently informative.

is that agents' posteriors about θ show no dependence on the prior mean, so a change in the quality of private information σ_η does not imply a common change of agents' beliefs towards or away from this prior mean. We discuss this issue more thoroughly in Section 4.1.

Although the prior is improper, the posterior is well defined. We obtain it by studying the distribution

$$\Pr(\theta \leq y \mid \theta_i) = \Pr(\theta_i - \sigma_\eta \eta_i \leq y \mid \theta_i) = 1 - F_\eta\left(\frac{\theta_i - y}{\sigma_\eta}\right)$$

Thus,

$$\theta \mid \theta_i \sim 1 - F_\eta\left(\frac{\theta_i - \theta}{\sigma_\eta}\right).$$

2.2 Specific Examples

The environment presented in the previous section nests a number of specific regime change games. In this section, we provide three examples: one on currency attacks, one on debt rollover, and a bank run.

2.2.1 Currency attack - Morris and Shin (1998)

We present a version of the classical Morris and Shin (1998) currency attack model. This model is a particular instance of the more general regime change model studied by Angeletos and Pavan (2013) once one abstracts from the signaling aspect of the game in their paper.

The economy features a central bank and a continuum of speculators. The policy maker decides on whether to maintain a particular exchange rate peg ($\mathcal{R} = 0$) or to abandon it ($\mathcal{R} = 1$). The speculators decide on whether to attack the peg by short selling the currency ($a_i = 1$) or to abstain ($a_i = 0$), with the aggregate action given by $A = \int_i a_i di$. Agent payoffs depend on a variable $\theta \in \mathbb{R}$, which describes the country's fundamentals.

The currency attack is successful whenever $A > g(\theta)$ for some threshold function $g(\theta)$. If the attack is successful, the currency is allowed to float at rate $f(\theta)$, where $f(\cdot)$ is an increasing function. Otherwise, the peg is maintained at e^*. The payoff of an attacker is $e^* - f(\theta) - t > 0$ if the attack is successful and $-t < 0$ if the attack fails, where t represents the cost of attacking the peg. An agent who does not attack has a payoff of 0 in either regime. Therefore, in this set-up, net payoffs are given by $U(\theta) = -t$ and $D(\theta) = e^* - f(\theta) - t$.[7]

2.2.2 A rollover game

The economy lasts for two dates $t = 0$ and $t = 1$. A financial institution enters $t = 0$ with liabilities in the form of expiring collateralized short-term debt contracts and an investment in long-term loans that mature only at $t = 1$. There is a continuum of creditors, each of whom holds a unit of this

[7] We also implicitly assume that $t < e^* - f(\theta)$, $\forall\theta$, so that $D(\theta) > 0$.

contract. Creditors choose whether to roll over their short-term contracts ($a_i = 0$) or refuse to roll over ("run", $a_i = 1$).

In the case of a refusal to roll over, a creditor receives a pre-determined fixed value of r. An agent that rolls over is promised a repayment of R at $t = 1$. The institution can resell any fraction α of its portfolio at a price of αr at $t = 0$ to meet repayment requests. If long-term loans are allowed to mature and are properly monitored (regime $\mathcal{R} = 0$), they can generate a per-loan payoff of $Y > R$ with probability $p_H(\theta)$, and zero otherwise. If no monitoring is done (regime $\mathcal{R} = 1$), they pay out Y with probability $p_L(\theta) < p_H(\theta)$. Both $p_L(\theta)$ and $p_H(\theta)$ are increasing and differentiable, with $p_H'(\theta) \geq p_L'(\theta)$, and $p_L(\theta) R < r < p_H(\theta) R$. An insider in the financial institution can monitor any amount of loans at a fixed cost of C. Incentive compatibility thus requires

$$C \leq (p_H(\theta) - p_L(\theta))(Y - R)(1 - A). \tag{2}$$

We define $g(\theta) = 1 - C(Y - R)^{-1}[p_H(\theta) - p_L(\theta)]^{-1}$ as the critical threshold, which is increasing in θ. Therefore, the net payoff differentials are given by $U(\theta) = r - p_H(\theta) R$ and $D(\theta) = r - p_L(\theta) R$, so $U(\theta)$ and $D(\theta)$ both vary with the fundamental.[8]

The main components of this example are that the liquidation of long-term projects is inefficient, and creditor decisions are strategic complements. These features are standard in debt-rollover and bank-run environments.

2.2.3 Bank run - Goldstein and Pauzner (2005)

We present a simplified version of the Goldstein and Pauzner (2005) model of demand deposits and bank runs, in which we disregard individual heterogeneity over discount factors.

A continuum of agents hold demand deposit contracts in a bank. The contract allows each agent to demand a fixed repayment $r > 1$ now (run, $a_i = 1$) or wait ($a_i = 0$). The bank has invested in a scalable long-term project that delivers 1 unit of a consumption good if liquidated now. Otherwise, if held until maturity, it delivers R units with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$.

The bank follows a sequential service constraint and fails ($\mathcal{R} = 1$) whenever it runs out of resources to repay early withdrawers at par, i.e., $Ar > 1$. In that case, agents that withdraw early are repaid r with probability $\frac{1}{Ar}$ and 0 otherwise, while agents that wait are paid 0. In case the bank survives ($\mathcal{R} = 0$), agents that withdraw early are repaid r with probability 1, while agents that wait are residual claimants on the future output, i.e., they are paid $\left(\frac{1-Ar}{1-A}\right) R$ with probability $p(\theta)$ and 0 otherwise.

We can then define $g(\theta) = \frac{1}{r}$, $U(\theta, A) = r - \left(\frac{1-Ar}{1-A}\right) R p(\theta)$ and $D(\theta, A) = \frac{1}{A}$. Note that these payoff differentials depend on both θ and A, which differs from our framework in Section 2.1, which assumes a dependence on θ only. However, as we show in Section 4.2, our main results carry through

[8] This is a feature that plays a central role in the analysis of the role of information quality. In addition, to anticipate our main result from Section 3 as applied to this example, note that even though $|U'(\theta)| > |D'(\theta)|, \forall \theta$, it does not imply that condition (11) holds.

with a dependence on both θ and A.

2.3 Equilibrium

We first define the Bayesian Nash Equilibrium (BNE) for the regime change game. We then characterize the equilibria under two assumptions that ensure the existence of upper and lower dominance regions. Under these assumptions, we show that the game has a unique BNE, and furthermore, that this equilibrium is in monotone strategies, a standard result from the global games literature (Morris and Shin (2003)).

Definition 1. A Bayesian Nash Equilibrium (BNE) for the regime change game consists of a strategy $a : \Theta \to \{0, 1\}$ and a fraction $A : \Theta \to [0, 1]$ of agents that play $a_i = 1$ s.t.

1. $a(\theta_i)$ is an individually optimal decision which sets:

$$\begin{cases} a(\theta_i) = 1, & if\ E_{\theta_i}[\pi(\theta, A(\theta))] > 0, \\ a(\theta_i) \in \{0,1\}, & if\ E_{\theta_i}[\pi(\theta, A(\theta))] = 0, \\ a(\theta_i) = 0, & if\ E_{\theta_i}[\pi(\theta, A(\theta))] < 0; \end{cases}$$

2. $A(\theta) = E_\theta[a(\theta_i)]$.

A monotone strategy is such that $a(\theta_i) = 1$ if, and only if, $\theta_i < \theta^*$, for some $\theta^* \in \mathbb{R}$, which we refer to as the strategic threshold.

We will characterize the equilibrium under the following assumptions:

A1. There exists a $\underline{\theta}$ such that $\forall \theta \leq \underline{\theta}$, $g(\theta) \leq 0$ with a strict inequality for $\theta < \underline{\theta}$.

A2. There exists a $\overline{\theta}$ such that $\forall \theta \geq \overline{\theta}$, $g(\theta) = 1$.

A3. For every $\sigma_\eta \in (0, \overline{\sigma}]$, $A \in [0, 1]$, and $\theta \in [\underline{\theta}, \overline{\theta}]$, $E_{\theta_i}[\pi(\theta, A)]$ and $E_{\theta_i}[\hat{\pi}_\theta(\theta, A)]$, in which

$$S_0(\theta, A) \equiv \begin{cases} \left|U'(\theta)\right| & , if\ A \geq g(\theta) \\ \left|D'(\theta)\right| & , if\ A < g(\theta) \end{cases},$$

exist and are finite.

(A1) ensures the existence of a lower dominance region, which is a region in which the state is so low that in a perfect information benchmark, playing $a_i = 1$ becomes a strictly dominant action. (A2) ensures that an upper dominance region exists: for a sufficiently high state θ, choosing $a_i = 0$ is a strictly dominant action. In principle, both dominance regions can be made arbitrarily small.[9] (A3) consists of two simple integrability assumptions.

Proposition 1 below, characterizes the unique equilibrium of this game under these assumptions.

[9] Note that (A1) and (A2) are sufficient conditions for the existence of dominance regions. Our results will go though for any alternative assumptions on model primitives that induce dominance regions.

Proposition 1. *Suppose that (A1), (A2), and (A3) hold. Then every BNE in monotone strategies of this economy can be described by a unique threshold* θ^**, which solves*

$$\int_0^1 \pi\left(\theta^* - \sigma_\eta F_\eta^{-1}(A), A\right) dA = 0. \tag{3}$$

such that each agent attacks when observing a signal below θ^* *and refrains from attacking when observing a signal above* θ^**. Furthermore, this equilibrium is the (essentially) unique BNE of the game.*

Proof. See Appendix. □

3 Understanding the role of information quality

We now turn to our main question: how information quality influences the equilibrium outcome in a regime-change game. To reduce the level of abstraction and for expositional clarity we frame this discussion in the context of our main motivating example on debt rollover even though the results we present apply more generally.

In the case of a financial firm, such as a bank, we seek to understand how information quality affects its stability by changing the set of fundamentals for which rollover crises occur and lead to its failure. We characterize a necessary and sufficient condition that is centrally related to payoff sensitivities and ensures that a decrease in the quality of individual information about the state θ (the firm's fundamentals) increases the set of fundamentals over which the financial institution fails (regime $\mathcal{R} = 1$ is realized). Given the diffuse prior assumption, these effects are consequences of a pure increase in the variance of beliefs, without any driving forces originating from changes in the mean beliefs, as would be the case with an informative prior. In Section 4.1, we extend our analysis to this second case.

In the unique equilibrium, two cutoffs are central to understanding the behavior of the economy. The first one is the strategic cutoff itself. Agents choose to run ($a_i = 1$) if and only if their signal about the fundamental state θ is worse than θ^*. As a consequence, the actual share of agents running upon the realization of any state θ is given by

$$A(\theta) = F_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right). \tag{4}$$

The lower the state, the higher the share of agents who receive a signal below their strategic cutoff and choose to run. The cutoff state in which the bank is on the verge of failure is thus given by θ^f, which satisfies

$$A\left(\theta^f\right) = F_\eta\left(\frac{\theta^* - \theta^f}{\sigma_\eta}\right) = g\left(\theta^f\right). \tag{5}$$

We refer to this second cutoff θ^f as the failure cutoff in the case of debt rollover or, more generally, as the regime-change cutoff. Figure 2 illustrates its equilibrium determination. Notice that there is a wedge between the two cutoffs, as equation (6) below shows.

$$\theta^{*}=\theta^{f}+\sigma_{\eta}F_{\eta}^{-1}\left(g\left(\theta^{f}\right)\right) \tag{6}$$

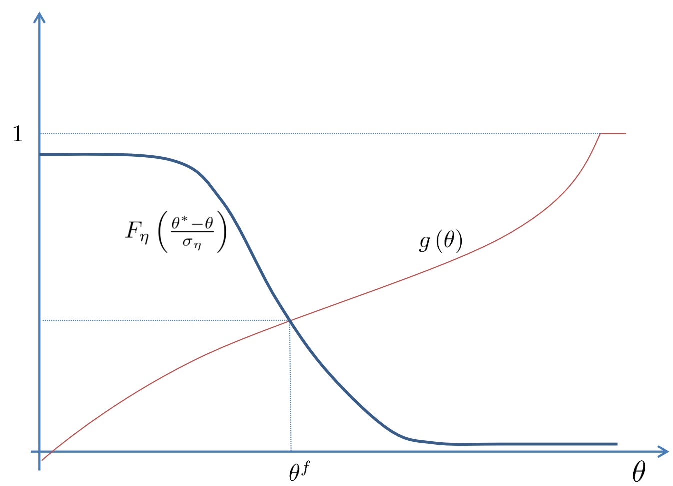


Figure 2: Equilibrium determination of the failure cutoff given strategies.

To understand the behavior of the equilibrium of the economy, it is necessary to study an agent who has received the cutoff signal, that is, an individual for which $\theta_i=\theta^*$. This agent has posterior beliefs about the underlying state given by $\theta \mid \theta^* \sim 1-F_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)$ and is indifferent between running ($a_i=1$) and rolling over ($a_i=0$). Additionally, this agent believes that the bank survives with probability $\Pr_{\theta^*}\left[\theta>\theta^f\right]\equiv F_\eta\left(\frac{\theta^*-\theta^f}{\sigma_\eta}\right)=g\left(\theta^f\right)$. Notice that $g\left(\theta^f\right)$ gains an important equilibrium interpretation as the probability of successful rollover (status quo survival, more generally), as perceived by the marginal agent.

Additionally, the indifference of this marginal agent implies that

$$\int_{-\infty}^{\theta^f} D(\theta)\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)d\theta=\int_{\theta^f}^{+\infty}\left|U(\theta)\right|\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)d\theta, \tag{7}$$

which can also be written as

$$E_{\theta^*}\left[D(\theta)\mid\theta\le\theta^f\right]\Pr_{\theta^*}\left[\theta\le\theta^f\right]=E_{\theta^*}\left[\left|U(\theta)\right|\mid\theta>\theta^f\right]\Pr_{\theta^*}\left[\theta>\theta^f\right]. \tag{8}$$

The indifference condition can be summarized as the equalization of the expected net payoff from attacking relative to not attacking when the bank fails (on the left-hand side), and the expected net payoff in case the bank survives (on the right-hand side), which must exactly offset each other. Another informative way of writing equation (8) is to substitute for the failure and survival probabilities and rearrange terms to get:

$$g\left(\theta^f\right) = \frac{E_{\theta^*}\left[D\left(\theta\right) \mid \theta \leq \theta^f\right]}{E_{\theta^*}\left[\left|U\left(\theta\right)\right| \mid \theta > \theta^f\right] + E_{\theta^*}\left[D\left(\theta\right) \mid \theta \leq \theta^f\right]}. \tag{9}$$

Therefore, the probability of successful rollover according to the marginal lender is a function of the ratio of the expected payoff differentials in case of bank survival and failure.

We use equation (6) to write a function $\theta^*\left(\theta^f, \sigma_\eta\right)$ and equation (7) to define an auxiliary function, which is useful for understanding the impact of information quality, as summarized by σ_η, on the failure threshold:

$$\begin{aligned}\psi\left(\theta^f, \sigma_\eta\right) \equiv\ & E_{\theta^*\left(\theta^f,\sigma_\eta\right)}\left[U\left(\theta\right) \mid \theta > \theta^f\right] \Pr_{\theta^*\left(\theta^f,\sigma_\eta\right)}\left[\theta > \theta^f\right] + \\ & + E_{\theta^*\left(\theta^f,\sigma_\eta\right)}\left[D\left(\theta\right) \mid \theta \leq \theta^f\right] \Pr_{\theta^*\left(\theta^f,\sigma_\eta\right)}\left[\theta \leq \theta^f\right].\end{aligned} \tag{10}$$

The restriction $\psi\left(\theta^f, \sigma_\eta\right) = 0$ implicitly defines the failure cutoff as a function of information quality, σ_η. We study its partial derivatives ψ_{θ^f} and ψ_{σ_η}. It turns out that ψ_{θ^f} is always negative, and as a consequence, the direction of the change of θ^f is determined by the sign of ψ_{σ_η}, as indicated in Proposition (2) below.

Proposition 2. *Let θ^f be the threshold of the state θ below which regime change ($\mathcal{R} = 1$) occurs. Then $\frac{\partial\theta^f}{\partial\sigma_\eta} > 0$ if, and only if,*

$$E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] < 0, \tag{11}$$

where θ^* is the unique threshold defined in Proposition 1, and $S\left(\theta, \theta^f\right) \equiv \begin{cases} \left|U'\left(\theta\right)\right| & , if\, \theta \geq \theta^f \\ \left|D'\left(\theta\right)\right| & , if\, \theta < \theta^f \end{cases}$.

Additionally, $\frac{\partial\theta^f}{\partial\sigma_\eta} = 0$ if, and only if, $E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] = 0$.

Proof. See Appendix. □

Condition 11 represents a covariation between the sensitivity in the marginal agent's payoff differentials and deviations of θ from θ^f. We can rewrite it as

$$\frac{E_{\theta^*}\left[\left|U'\left(\theta\right)\right|\left(\theta - \theta^f\right) \mid \theta > \theta^f\right]}{E_{\theta^*}\left[\left|U\left(\theta\right)\right| \mid \theta > \theta^f\right]} < \frac{E_{\theta^*}\left[\left|D'\left(\theta\right)\right|\left(\theta^f - \theta\right) \mid \theta \leq \theta^f\right]}{E_{\theta^*}\left[D\left(\theta\right) \mid \theta \leq \theta^f\right]}. \tag{12}$$

We can interpret this version of our main condition in the following way. It represents a measure of the responsiveness of payoffs to the fundamentals, behaving similarly to two elasticity terms. Holding the cutoff fixed, if the payoff differential in the case of bank failure becomes more elastic, condition (11) holds more easily. This elasticity interpretation is precise in the noiseless limit as shown in Proposition 3.

A decrease in information quality increases the expected net payoff in the case that the financial institution fails (regime change) but also lowers the (negative) expected net payoff in the case of successful rollover (status quo survival). Whenever the payoff difference in the case of bank failure is more sensitive than the payoff difference in the case of successful rollover, in the sense of condition (11) or (12) holding, a decrease in information precision causes an agent's net payoff from running versus rolling over to strictly increase. As a consequence, condition (11) ensures that when information quality decreases, the equilibrium marginal agent must change towards an agent that observes a higher signal realization. Thus, the failure cutoff θ^f increases and bank failure becomes more common.

To provide a clearer interpretation of payoff sensitivities, we consider several special cases in the next section.[10]

3.1 Special cases

A clear example in which a decrease in information quality decreases financial stability (i.e., it expands the set of fundamentals θ for which regime change occurs) is the situation in which θ influences the determination of the failure threshold and the net payoff in the case of bank failure, while the net payoff in the case of successful rollover is fixed. In terms of our notation, this is represented by a situation in which $g(\theta)$ and $D(\theta)$ show dependence on the state, but $U(\theta)$ is independent of θ. We summarize this and other possible configurations below.

Corollary 1.

1. *Suppose that* $U(\theta) = U$, $\forall\theta$ *and* $D(\theta)$ *is strictly decreasing everywhere. Then* $\frac{\partial\theta^f}{\partial\sigma_\eta} > 0$.

2. *Suppose that* $D(\theta) = D$, $\forall\theta$ *and* $U(\theta)$ *is strictly decreasing everywhere. Then* $\frac{\partial\theta^f}{\partial\sigma_\eta} < 0$.

3. *Suppose that* $U(\theta) = U$ *and* $D(\theta) = D$, $\forall\theta$. *Then* $\frac{\partial\theta^f}{\partial\sigma_\eta} = 0$.

Proof. See Appendix. □

This result is intuitive because with no sensitivity in the net payoff in the case of successful rollover, lenders are exposed only to downside risks – the possibility that they face a financial institution with very low fundamentals θ. Therefore, decreasing the precision of their beliefs can only increase their net payoff from running versus rolling over. On the other hand, with no sensitivity in the net payoff in case of bank failure, lenders are exposed only to upside risks - the possibility that they face a financial institution with high fundamentals θ. In that case, a decrease in precision can only decrease their expected net payoff.

[10] For an additional interpretation of condition (11) as a comparison between the variance sensitivities of two options that comprise lender net payoffs, please consult the working paper version of this article (Iachan and Nenov, 2013).

Applying this result to the canonical currency attack model of Morris and Shin (1998), reintroduced in section 2.2.1, we notice that risk is concentrated on one side: agents that short the currency have large payoff gains if fundamentals are low without suffering increasing losses in the region of fundamentals where the peg survives. As a consequence, lower information quality increases the expected gains from attacking the currency and facilitates currency crises by increasing θ^f.

Another way to analyze the importance of the relative sensitivity of the two net payoff differentials to the effect of information quality on the regime change cutoff θ^f is to consider a class of net payoff functions for which one of the two payoff differentials is more sensitive than the other at any distance from that cutoff. The definition of such payoff functions is similar to Albagli, Hellwig, and Tsyvinski (2011):

Definition 2. For a given regime change threshold θ^f, the net payoff function π is dominated by upside risks at θ^f if $\left|U'\left(\theta^f + x\right)\right| > \left|D'\left(\theta^f - x\right)\right|$ for all $x \geq 0$. The net payoff function π is dominated by downside risks at θ^f if $\left|U'\left(\theta^f + x\right)\right| < \left|D'\left(\theta^f - x\right)\right|$ for all $x \geq 0$.

Therefore, for functions dominated by upside risks, marginal changes in the payoff in the case of status quo survival always dominate marginal changes in the payoff in the case of regime change, as we move away from the regime change threshold. The opposite holds for functions dominated by downside risks. Hence, if we look at a mean preserving spread of a distribution centered at θ^f, the expected net payoff will decrease for a function dominated by upside risks. The effect is the opposite for a function dominated by downside risks.

Given this definition, we have the following illustrative corollary to Proposition 2.

Corollary 2. *Suppose that η_i follows a symmetric distribution, i.e. $f_\eta(x) = f_\eta(-x)$, $\forall x \in \mathbb{R}$ and $f_\eta(x)$ is (weakly) decreasing in x for $x \in \mathbb{R}^+$.*

1. *If π is dominated by upside risks at θ^f and $g\left(\theta^f\right) \geq \frac{1}{2}$, then $\frac{\partial \theta^f}{\partial \sigma_\eta} < 0$.*

2. *If π is dominated by downside risks at θ^f and $g\left(\theta^f\right) \leq \frac{1}{2}$, then $\frac{\partial \theta^f}{\partial \sigma_\eta} > 0$.*

Proof. See Appendix. □

This result confirms the intuition for the result of Proposition 2. If the payoff differential in the case of regime change is more sensitive to the fundamentals than its counterpart and the marginal financial institution is sufficiently fragile (in the sense that it requires less than half of the agents to run for the institution to fail), then a lender who is originally indifferent between running and rolling over has a strictly higher expected payoff from running given a marginal decrease in precision. As a consequence, the failure cutoff moves up and institutions fail for a larger set of fundamentals.

3.2 A limit result

The limit of a noiseless economy is also informative about how the regime change threshold's response to information quality depends on the relative payoff sensitivities in economies with sufficiently small uncertainty.

Proposition 3. *The limit noiseless economy obtained by taking the limit of a sequence of economies for which $\sigma_\eta \to 0$, satisfies*

$$U\left(\theta^f\right) g\left(\theta^f\right) + D\left(\theta^f\right)\left(1 - g\left(\theta^f\right)\right) = 0. \tag{13}$$

and $\theta^* = \theta^f$. Furthermore,

$$\lim_{\sigma_\eta \to 0} \frac{\partial \theta^f}{\partial \sigma_\eta} > 0 \iff \left|\frac{U'\left(\theta^f\right)}{U\left(\theta^f\right)}\right| < \left|\frac{D'\left(\theta^f\right)}{D\left(\theta^f\right)}\right| h\left(g\left(\theta^f\right)\right), \tag{14}$$

in which $h : [0,1] \to \mathbb{R}_{++}$ satisfies $h\left(g\left(\theta^f\right)\right) = -\frac{\lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma_\eta} | \theta < \theta^f\right]}{\lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma_\eta} | \theta \geq \theta^f\right]}$. If $\eta \sim N(0,1)$, then h is strictly decreasing in $g\left(\theta^f\right)$.

Proof. See Appendix. □

Proposition (3) shows that two factors determine whether a change in information quality favors the status quo or regime change in economies with small amounts of noise. The first factor is clearly the relation between the two net payoff elasticities, which in this limit are simply evaluated at the regime-change cutoff itself. A sufficiently high payoff elasticity in the case of regime change relative to regime survival implies that less precise information favors regime change.

The second factor is related to the value of the critical threshold $g\left(\theta^f\right)$, suggesting that the effect of information quality on the regime change threshold can also depend on exogenous parameters that shift it. Let us illustrate this possibility with a simple example based on our debt rollover motivation. Remember that $g(\theta)$ represents the share of agents that needs to run in order for the institution with fundamentals given by θ to fail. Therefore, $g(\theta)$ can also be interpreted as the resilience of that institution. This degree of resilience can depend on aggregate conditions in asset and credit markets, for example. We can imagine two situations: one in a relatively stable financial environment, where $g(\theta) = \Gamma_h$, $\forall \theta$, in which a high share of agents needs to run to induce failure, and another in a more fragile environment where $g(\theta) = \Gamma_l < \Gamma_h$, $\forall \theta$, so a run by fewer agents leads to failure. Proposition 3 then shows that a decrease from Γ_h to Γ_l (for example, due to a deterioration in credit market conditions) adds to the effects of lower information quality. Therefore, in that context, holding payoff sensitivities fixed, condition (14) holds more easily in an economy with worse credit market conditions. Intuitively, an increase in the level of fragility in the economy acts as an amplifier of changes in information quality.[11]

[11] In this example, Γ_i serves as an exogenous shifter of the threshold $g(\theta)$, which is constant for all values of θ.

4 Extensions

4.1 Informative Priors and Public Information

So far, we have studied the effect of changes in information quality in the case of a diffuse prior. In that case, the marginal agent believes that the fraction of the population attacking the status quo is distributed uniformly on the $[0,1]$ interval[12] irrespective of the quality of information the agent uses to form his posterior belief over θ. As a consequence, an increase in the precision of private information does not facilitate coordination. This is generally not the case under an informative prior, as in that case the source of information (public versus private signal) matters for agent actions beyond the overall quality of information by affecting the agents' ability to coordinate (Morris and Shin, 2002, Angeletos and Pavan, 2004, Angeletos and Pavan, 2007).

In this section, we investigate how changes in information quality affect regime-change in the presence of an informative public signal (a non-diffuse prior). As we show, in that environment, our main result carries over, but there are additional effects arising from the position of the prior mean relative to the failure cutoff.

We conduct our analysis under the assumption of a normally distributed prior and private signals. In particular, we assume that $\epsilon \sim N(0,1)$ and $\eta \sim N(0,1)$. Given these distributions, standard results from Bayesian learning imply that the posterior belief about θ for an agent that observes a signal θ_i is normally distributed with a posterior mean $\mu(\theta_i, \rho)$ given by

$$\mu(\theta_i, \rho) = \rho^2 \theta_i + \left(1 - \rho^2\right) \theta_0$$

and a posterior variance σ given by

$$\sigma^2 = \frac{\sigma_\epsilon^2 \sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2},$$

where

$$\rho^2 \equiv \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2} = \frac{\sigma^2}{\sigma_\eta^2} \in [0,1]$$

is the weight that an agent puts on his private signal. Notice that ρ is directly related to the information content of public relative to private information, and in that sense, it is not related to the overall quality of information. In particular, an increase in ρ holding σ fixed, corresponds to a decrease in the ratio $\frac{\sigma_\eta}{\sigma_\epsilon}$, that is, a deterioration in the quality of public relative to private information.

To understand the role of information quality in the case of an informative prior, it is particularly useful to consider changes in the posterior variance σ^2 holding ρ and θ_0 constant. Such a decomposition is similar to that in Angeletos and Pavan (2004) and allows us to discuss information

These results can be generalized for other cases involving shifters in the cutoff function $g(\theta)$. It can also be shown to be a possible source of non-monotonicity in the effects of information quality depending on the initial value of θ^f (Iachan and Nenov, 2013).

[12] The agent has Laplacian beliefs, in the sense of Morris and Shin, 2003.

quality effects, holding the relative quality of public and private information constant. In turn, a change in the quality of public (σ_ϵ) or private (σ_η) information will have an effect through these *absolute information quality* (σ) and *relative information quality* (ρ) channels on the regime change cutoff θ^f.

As in the diffuse-prior model, we can characterize equilibria as the intersection of two conditions. The first condition is related to the behavior of the marginal agent, who knows that regime change occurs for $\theta < \theta^f$ and observes a private signal at the strategic cutoff θ^* (so he has posterior beliefs centered at $\mu = \rho^2\theta^* + \left(1-\rho^2\right)\theta_0$). That agent is indifferent between attacking and not attacking the status quo. The second condition is given by the mass of agents who attack given that θ equals the regime change threshold θ^f.

We consider the effects of absolute and relative information quality on the regime change threshold in a neighborhood of the economy where public information has zero precision.

Proposition 4. *As private information becomes arbitrarily more precise than public information, i.e., $\rho \to 1$, the effects of absolute information quality (σ) and relative information quality (ρ) on the equilibrium failure threshold are given by*

$$\lim_{\rho\to 1}\frac{\partial\theta^f}{\partial\sigma} > 0 \iff E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta-\theta^f\right)\right] < 0 \tag{15}$$

and

$$\lim_{\rho\to 1}\frac{\partial\theta^f}{\partial\rho} > 0 \iff \theta_0 > \frac{\theta^f+\theta^*}{2} \tag{16}$$

where $\theta^* = \theta^f + \frac{\sigma}{\rho}\Phi^{-1}\left(g\left(\theta^f\right)\right)$, *and* $S\left(\theta,\theta^f\right) \equiv \begin{cases} \left|U'(\theta)\right| & , if\ \theta \geq \theta^f \\ \left|D'(\theta)\right| & , if\ \theta < \theta^f \end{cases}.$

Proof. See Appendix. □

To interpret this result, note first that the effect of absolute information quality $\frac{\partial\theta^f}{\partial\sigma}$ depends on the exact same condition as that in Section 3, namely on the relative sensitivity of net payoff differentials.

The effect of relative information quality $\frac{\partial\theta^f}{\partial\rho}$ on the other hand depends only on the position of the prior mean θ_0. If the prior mean is low, i.e., $\theta_0 < \frac{\theta^f+\theta^*}{2}$, an increase in ρ (a decrease in the relative precision of public information) moves the posterior mean away from the prior. This acts as a rightward shift in the belief distribution. As a result, the regime change cutoff falls. The opposite effect occurs when the prior mean is high, i.e., $\theta_0 > \frac{\theta^f+\theta^*}{2}$.

An even stronger result is obtained when one considers the limit of both $\sigma \to 0$ and $\rho \to 1$. In that case, for the effect of absolute information quality we recover the result from Proposition 3, whereas for the effect of relative information quality we obtain

$$\lim_{\sigma\to 0,\rho\to 1}\frac{\partial\theta^f}{\partial\rho} > 0 \iff \theta_0 > \theta^f \tag{17}$$

Therefore, in the limit, the effect of relative information quality depends on the position of the prior mean relative to the regime change cutoff.

A result similar to Proposition 4 is obtained away from the limit where ρ equals one, as we show in the online supplement. In particular, the effect of absolute information quality depends on a modified version of our main condition (11), which includes an additional effect from a change in σ apart from the effect through payoff sensitivities. Similarly to the effect of changes in ρ, this additional effect also depends on the position of the prior mean relative to the regime change cutoff.[13]

4.2 Payoff Externalities in Addition to Regime Determination

In many important regime-change games, payoffs can be affected by the actions of others beyond the determination of the equilibrium regime. For instance, in a typical bank run environment, if the bank is liquidated, the payoff for each withdrawer depends on the mass of agents choosing each action. In this section, we show that the results from Section 3 are maintained when we consider more general net payoff differentials, which also depend on the fraction of agents that play $a_i = 1$. We look at payoffs of the form:

$$\pi(\theta, A) \equiv \begin{cases} U(\theta, A) & , if\ A \leq g(\theta) \\ D(\theta, A) & , if\ A > g(\theta). \end{cases} \tag{18}$$

Therefore, agents' actions are allowed to exert an additional externality by influencing not only the regime outcome $\mathcal{R}$, but also other agents' payoffs in the case of a particular regime realization. We maintain all previous assumptions about the function $g(\theta)$ and additionally assume that $U(\theta, A) < 0$ and $D(\theta, A) > 0$, and that both net payoffs are twice continuously differentiable in (θ, A) and non-increasing in θ.[14]

We have the following

Proposition 5. *Let θ^f be the cutoff of fundamentals for which regime change occurs. Then, $\frac{\partial \theta^f}{\partial \sigma_\eta} > 0$ if, and only if,*

$$E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] < 0 \tag{19}$$

[13] For the case where net payoff differentials are not sensitive to changes in fundamentals, the effects of both absolute and relative information quality depend only on the position of the prior mean relative to the regime change cutoff. These effects are directly related to existing results in the global games literature (Metz (2002), Bannier and Heinemann (2005), and Angeletos, Hellwig, and Pavan (2007)). In recent work, Vives (2014) obtains a similar result in a model of debt rollover.

[14] Note that these payoff assumptions are not sufficient to guarantee a unique equilibrium in cutoff strategies. A simple possibility is to assume that U and D are non-decreasing in A and have negative cross-partials, $U_{\theta,A} < 0$ and $D_{\theta,A} < 0$, making the game supermodular in $(\theta, -A)$. The steps of the proof would then follow Proposition 1. Weaker assumptions on the effects of A such as those in Goldstein and Pauzner (2005) also lead to uniqueness. Even without a unique equilibrium, the comparative static in Proposition 5 still holds around every equilibrium cutoff θ^f.

where $\theta^* = \theta^f + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right)$ *and*

$$S\left(\theta, \theta^f\right) \equiv \begin{cases} \left|U_\theta\left(\theta, F_\eta\left(\frac{\theta^f - \theta}{\sigma_\eta} + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)\right| & , if\, \theta \geq \theta^f \\ \left|D_\theta\left(\theta, F_\eta\left(\frac{\theta^f - \theta}{\sigma_\eta} + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)\right| & , if\, \theta < \theta^f \end{cases}$$

Proof. See Appendix. □

This result is essentially the same as that in Proposition 2, and one can again re-write the condition obtained as a comparison of relative payoff sensitivities. Therefore, the effect of more precise information still depends on the relative responsiveness of net payoff differentials to fundamentals.

The result above can be applied directly to the study of information quality in bank run models based on Diamond and Dybvig (1983) and Goldstein and Pauzner (2005). In particular, Condition 19 fails to hold in the original formulation of Goldstein and Pauzner (2005) and, as a consequence, an improvement in information quality makes bank runs more likely by increasing the failure threshold.

Corollary 3 (Corollary to Proposition 5)**.** *Consider the bank run model as described in Section 2.2.3. Then,* $E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] > 0$ *and* $\frac{\partial \theta^f}{\partial \sigma_\eta} < 0$.

Proof. See Appendix. □

The reason for this result is that depositors who do not withdraw early in that model are always residual claimants on the non-liquidated fraction of a long-term project. The quality of this project depends on the fundamentals, while its liquidation value does not. This generates sensitivity for the net payoff conditional on bank survival, while the net payoff in the case of failure remains independent of the fundamentals.

4.3 Welfare and Policy

The seminal contribution of Carlsson and van Damme (1993) illustrates that small amounts of private information can ensure unique equilibrium predictions in environments where strategic complementarities create a force towards multiplicity.[15] Their abstract 2-player 2-action environment has a particular feature shared by most other global games applications: equilibria, in the common knowledge benchmark, can be Pareto ranked and inefficiencies originate from the inability to coordinate on a superior alternative.

In settings with a continuum of players and dispersed information, such as the ones we analyze, coordination failures manifest themselves in two ways: the regime switching cutoff is generically at an inefficient level; and agents that receive extreme signals fail to take the action that is optimal given the equilibrium regime. Agents fully incorporate this latter possibility of individual mistakes in their decisions, but the former generates an externality that policy could target.

[15] Frankel, Morris, and Pauzner (2003) generalize these results to multi-agent multi-action games with strategic complementarities.

We formally describe these results in the online supplementary material. We consider a constrained efficiency benchmark as in Angeletos and Pavan (2007) in which a planner can mandate that agents play strategies that are not privately optimal but cannot transmit any information across agents. The planner takes into account the relationship between the strategic cutoff and the regime-change cutoff and so internalizes both the direct effect of the strategic cutoff θ^* on individual welfare as well as its effect through the determination of the regime-change cutoff θ^f (the externality). The sign of the externality is the same as the effect of θ^f on welfare. For instance, in any environment in which decreasing θ^f increases welfare, agents would choose individual strategies that too frequently lead to regime-change from a social perspective.

As a consequence of the underlying externality, there is a natural role for policy in these environments. There is also a natural benchmark to evaluate its effects: whether a policy mitigates or exacerbates the underlying inefficiency is directly related to the direction of the change it induces on θ^f. This provides a justification to our positive focus on comparative statics with respect to the regime change cutoff in the previous sections.

Any policy that induces a change in the information structure has both a direct and an indirect effect on equilibrium welfare. For instance, a policy that ultimately makes private information more precise has a positive direct impact on welfare, as the private value of information is positive because agents can better tailor their actions to the underlying state. The indirect effect is related to the change in the externality and is fully summarized by the change in the regime-change cutoff.

4.4 Information acquisition

We have focused on an exogenous information structure with a very tractable notion of information quality. Agents are endowed with a particular information generation technology, and we study the equilibrium consequences of changes in information quality. We have abstracted from the tradeoffs that agents would face when choosing among alternative information structures, which is the main focus of the growing literature on endogenous information acquisition (Hellwig and Veldkamp, 2009, Myatt and Wallace, 2012, Colombo, Femminis, and Pavan, 2013, Yang, 2013, Szkup and Trevino, 2013, among others).[16] Our analysis is complementary to this literature. It can be particularly useful in helping map changes in equilibrium information structures, which could originate from variations in exogenous costs or from policy interventions, into changes in equilibrium regime determination.

The earlier literature on endogenous information acquisition has focused on restricted information structures, where agents choose among alternative distributions and precisions of signals in a costly way. Following the "rational inattention" approach of Sims (2003), a promising new branch of research initiated by Yang (2013) allows agents to choose any arbitrary signal generation structure

[16] As the class of games we consider includes strategic complementarities, general results from the literature hold in our setting. For example, agents would prefer to observe and act more intensely on signals that are more correlated (Hellwig and Veldkamp (2009)). There may also be multiple information structures that are consistent with an equilibrium. For regime-change games, which display the strong complementarities in the sense of Angeletos and Pavan (2004), an over-reliance on a correlated signal may end up restoring equilibrium multiplicity.

subject to a cost that is proportional to the reduction of entropy it induces. In other words, agents can specifically tailor which events they become better informed about.

To extend our analysis in this direction, we propose the following exercise in the online supplementary material. We assume that a change in an information acquisition parameter or a change in equilibrium beliefs occurs in an arbitrary but continuous way. We still impose an additive signal structure, with noise that is independent from the fundamentals and i.i.d. across agents.[17] Then,

$$\eta_i \sim H(\eta_i, \alpha) \equiv \alpha F_{\eta,1}(\eta_i) + (1-\alpha) F_{\eta,0}(\eta_i).$$

By studying small perturbations away from $F_{\eta,0}$ using this mixture formulation, we describe the consequences of a change in the distribution of private noise towards an arbitrary $F_{\eta,1}(\eta_i)$. We provide a condition that incorporates the effects of payoff sensitivities in this context. In particular, if under $F_{\eta,1}$, extremely high realizations of η_i are very likely, the agent understands that a signal $\theta_i = \theta^*$ can still occur very frequently, even for low values of θ; that is, the agent puts more weight on payoffs from low realizations of θ as α increases.[18] This induces an increase in the strategic threshold, caused by an effect that depends on the net payoff sensitivities.

5 Concluding comments

This paper examines how information quality affects the unique equilibrium outcome of a global game of regime change. We show that a deterioration in information quality leads to regime instability whenever the net payoff in the case of regime change is more sensitive to the fundamentals than the net payoff in the case of the survival of the status quo. An externality is present in this class of environments, as agents do not internalize the effect of their individual strategies on the likelihood of regime change. Therefore, policies that influence information quality can be welfare-improving.

The model we analyze is general and involves flexible reduced-form payoff functions. When applied to the context of debt rollover, it takes a short-term debt contract as given. A natural direction for future research includes the explicit modeling of one particular financial contracting problem. Such an exercise may lead to additional restrictions on payoffs and a better understanding of the consequences of alternative policy interventions.

Furthermore, understanding the overall effectiveness of any policy, including disclosure policies, must ultimately take into account the strategic incentives of a regulator for releasing or withholding information. Such signaling issues bring forth concerns about possible policy traps (Angeletos, Hellwig, and Pavan (2007), Angeletos and Pavan (2013)), which should be incorporated in a thorough welfare evaluation of a particular disclosure policy. Lastly, questions about the dynamic revelation of information, which is naturally irreversible, emerge as an important step for future research.

[17] Yang (2013) shows that in environments with discontinuous payoffs, like the regime-change games we study, there is a continuum of equilibria. We avoid this issue with this particular approach.

[18] The condition also contains an effect originating from the change in the relationship between the strategic and the regime-change cutoffs as one moves across noise distributions.

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Appendix

A Proofs

Proof of Proposition 1

Proof. Let us first define $A\left(\theta,\tilde{\theta}\right) \equiv F_\eta\left(\frac{\tilde{\theta}-\theta}{\sigma_\eta}\right)$ as the fraction of agents that play $a_i = 1$ (attack) if the state is θ when agents follow a strategy with cutoff $\tilde{\theta}$. Secondly, we define

$$v\left(\theta_i,\tilde{\theta}\right) \equiv E_{\theta_i}\left[\pi\left(\theta, A\left(\theta,\tilde{\theta}\right)\right)\right] = \int_{-\infty}^{\infty} \pi\left(\theta, A\left(\theta,\tilde{\theta}\right)\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)d\theta$$

as the expected net payoff from playing $a_i = 1$ versus $a_i = 0$ for an agent that observes a signal θ_i and expects other agents to follow cutoff strategies with cutoff $\tilde{\theta}$, where f_η is the probability density function of F_η. Lastly, we define $\theta^f\left(\tilde{\theta}\right)$ implicitly by $A\left(\theta^f,\tilde{\theta}\right) = F_\eta\left(\frac{\tilde{\theta}-\theta^f\left(\tilde{\theta}\right)}{\sigma_\eta}\right) = g\left(\theta^f\left(\tilde{\theta}\right)\right)$. Note that θ^f is continuous and increasing in $\tilde{\theta}$ by the implicit function theorem. Also, by assumptions (A1) and (A2), $\theta^f \to \overline{\theta}$, as $\tilde{\theta} \to \infty$, and $\theta^f \to \underline{\theta}$, as $\tilde{\theta} \to -\infty$.

We first show several important properties of $v\left(\theta_i,\tilde{\theta}\right)$.

First, $v\left(\theta_i,\tilde{\theta}\right)$ is continuous and strictly decreasing in θ_i. Continuity follows, since

$$\pi\left(\theta, A\left(\theta,\tilde{\theta}\right)\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)$$

is continuous in θ_i and, by assumption (A3), $\pi\left(\theta, A\left(\theta,\tilde{\theta}\right)\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)$ is integrable, $\forall\theta_i \in \mathbb{R}$. To show strict monotonicity, notice that we can write $v\left(\theta_i,\tilde{\theta}\right)$ as

$$v\left(\theta_i,\tilde{\theta}\right) = \int_{-\infty}^{\theta^f\left(\tilde{\theta}\right)} D\left(\theta\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)d\theta +$$
$$+\int_{\theta^f\left(\tilde{\theta}\right)}^{\infty} U\left(\theta\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)d\theta = \int_{-\infty}^{\infty}\tilde{\pi}\left(\theta,\tilde{\theta}\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)d\theta$$

where $\tilde{\pi}\left(\theta,\tilde{\theta}\right) = \begin{cases} D\left(\theta\right) & ,\theta \le \theta^f\left(\tilde{\theta}\right) \\ U\left(\theta\right) & ,\theta > \theta^f\left(\tilde{\theta}\right)\end{cases}$. Then, $v\left(\theta_i,\tilde{\theta}\right)$ is strictly decreasing in θ_i, since $\tilde{\pi}\left(\theta,\tilde{\theta}\right)$ is decreasing in θ, and since an agent's posterior probability over events $\{\theta < y\}$, $1 - F_\eta\left(\frac{\theta_i-y}{\sigma_\eta}\right)$, is strictly decreasing in θ_i, i.e. an increase in θ_i leads to a shift in an agent's belief in a first-order stochastic dominance sense.

Secondly, $v\left(\theta_i,\tilde{\theta}\right)$ is continuous, and strictly increasing in $\tilde{\theta}$, for $\tilde{\theta} > \underline{\theta}$. Continuity follows, since both $D(\theta)\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)$ and $U(\theta)\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta_i-\theta}{\sigma_\eta}\right)$ are integrable, given (A3), and $\theta^f\left(\tilde{\theta}\right)$ is continuous in $\tilde{\theta}$. Monotonicity follows from differentiation of $v\left(\theta_i,\tilde{\theta}\right)$ with respect to $\tilde{\theta}$ and application of the Leibniz rule.

Next, we define the function $\tilde{v}\left(\tilde{\theta}\right) \equiv v\left(\tilde{\theta},\tilde{\theta}\right)$. The function is continuous, and strictly decreasing in $\tilde{\theta}$, for $\tilde{\theta} > \underline{\theta}$. Continuity follows from the continuity of $v\left(\theta_i,\tilde{\theta}\right)$. To show monotonicity, first note that for $\tilde{\theta}$, we have that $A\left(\theta,\tilde{\theta}\right) = F_\eta\left(\frac{\tilde{\theta}-\theta}{\sigma_\eta}\right)$, or inverting this function, $\theta = \tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)$. Hence, we can do a change of variables in $\tilde{v}\left(\tilde{\theta}\right)$ and rewrite it as

$$\tilde{v}\left(\tilde{\theta}\right) = \int_0^1 \pi\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A), A\right)\frac{1}{\sigma_\eta}f_\eta\left(F_\eta^{-1}(A)\right)\frac{\sigma_\eta}{f_\eta\left(F_\eta^{-1}(A)\right)}dA \tag{A.1}$$

or equivalently

$$\tilde{v}\left(\tilde{\theta}\right) = \int_0^{g\left(\theta^f\left(\tilde{\theta}\right)\right)} U\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA + \int_{g\left(\theta^f\left(\tilde{\theta}\right)\right)}^1 D\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA \tag{A.2}$$

Differentiating with respect to $\tilde{\theta}$, we get:

$$\begin{aligned}\frac{\partial\tilde{v}}{\partial\tilde{\theta}} &= \left[U\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\left(\tilde{\theta}\right)\right)\right)\right) - D\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\left(\tilde{\theta}\right)\right)\right)\right)\right]g_\theta\frac{\partial\theta^f}{\partial\tilde{\theta}} \\ &\quad + \int_0^{g\left(\theta^f\left(\tilde{\theta}\right)\right)} U'\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA + \int_{g\left(\theta^f\left(\tilde{\theta}\right)\right)}^1 D'\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA\end{aligned}$$

Each of these terms is non-positive, given the assumptions on $U(\theta)$ and $D(\theta)$, with at least one the three terms strictly negative, given the assumption that at least one of $U(\theta)$, $D(\theta)$, and $g(\theta)$ are strictly monotone in θ.

Given assumptions (A1) and (A2), $\theta^f \to \overline{\theta}$, as $\tilde{\theta} \to \infty$ and $\theta^f \to \underline{\theta}$, as $\tilde{\theta} \to -\infty$, so $g\left(\theta^f\left(\tilde{\theta}\right)\right) \to 0$ as $\tilde{\theta} \to -\infty$ and $g\left(\theta^f\left(\tilde{\theta}\right)\right) \to 1$ as $\tilde{\theta} \to \infty$. Furthermore, U and D are bounded, so $U(\theta) \geq \underline{U}$, $\forall\theta$ and $D(\theta) \leq \overline{D}$, $\forall\theta$. Therefore, $\int_0^{g\left(\theta^f\left(\tilde{\theta}\right)\right)} U\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA > \int_0^{g\left(\theta^f\left(\tilde{\theta}\right)\right)} \underline{U}dA$. Furthermore, $\int_0^{g\left(\theta^f\left(\tilde{\theta}\right)\right)} \underline{U}dA \to 0$ as $\tilde{\theta} \to -\infty$ and $\int_{g\left(\theta^f\left(\tilde{\theta}\right)\right)}^1 D\left(\tilde{\theta} - \sigma_\eta F_\eta^{-1}(A)\right)dA > 0$, $\forall\tilde{\theta}$, since $D(\theta) > 0$ for some θ and $D(.)$ is non-increasing. Hence, there is a $\tilde{\theta}_L$, such that $\tilde{v}\left(\tilde{\theta}\right) > 0$ for $\tilde{\theta} \leq \tilde{\theta}_L$. Similarly, there is a $\tilde{\theta}_H$, such that $\tilde{v}\left(\tilde{\theta}\right) < 0$ for $\tilde{\theta} \geq \tilde{\theta}_H$. Therefore,

$$\tilde{v}(\theta^*) = 0 \tag{A.3}$$

has a unique solution θ^*.

Note that θ^* describes the threshold for a monotone equilibrium, if, and only if,

$$v(\theta_i,\theta^*) < 0, \quad \forall\theta_i > \theta^* \tag{A.4}$$

and

$$v(\theta_i,\theta^*) > 0, \quad \forall\theta_i < \theta^*. \tag{A.5}$$

which follow from the properties of $v\left(\theta_i,\tilde{\theta}\right)$. Therefore, the solution to:

$$\int_0^1 \pi\left(\theta^* - \sigma_\eta F_\eta^{-1}(A), A\right)dA = 0 \tag{A.6}$$

describes the unique threshold for the monotone equilibrium.

To show that this is the only equilibrium of this game, we proceed by showing that the monotone strategy with cutoff θ^* survives a procedure of iterated deletion of strictly dominated strategies. Let $\underline{\zeta}_0 = -\infty$ and $\overline{\zeta}_0 = \infty$ and define recursively the sequences $\underline{\zeta}_{n+1} = \min\left\{\theta_i : v\left(\theta_i,\underline{\zeta}_n\right) = 0\right\}$ and $\overline{\zeta}_{n+1} = \max\left\{\theta_i : v\left(\theta_i,\overline{\zeta}_n\right) = 0\right\}$ for $n \geq 1$. First of all, with a slight abuse of notation, let $v\left(\theta_i,\underline{\zeta}_0\right) = \lim_{\tilde{\theta}\to-\infty} v\left(\theta_i,\tilde{\theta}\right)$. Given assumption (A1), $\theta^f \to \underline{\theta}$, as

$\tilde{\theta} \to -\infty$, so

$$v\left(\theta_i, \tilde{\theta}\right) = \int_{-\infty}^{\underline{\theta}} D(\theta) \frac{1}{\sigma_\eta} f_\eta \left(\frac{\theta_i - \theta}{\sigma_\eta}\right) d\theta + \int_{\underline{\theta}}^{\infty} U(\theta) \frac{1}{\sigma_\eta} f_\eta \left(\frac{\theta_i - \theta}{\sigma_\eta}\right) d\theta$$

Given that U is bounded, so $U(\theta) \geq \underline{U}$, $\forall \theta$, it follows that $\int_{\underline{\theta}}^{\infty} U(\theta) \frac{1}{\sigma_\eta} f_\eta \left(\frac{\theta_i - \theta}{\sigma_\eta}\right) d\theta > \overline{U}\left(1 - F_\eta\left(\frac{\theta_i - \underline{\theta}}{\sigma_\eta}\right)\right)$. Since this latter term vanishes as $\theta_i \to -\infty$ and $\int_{-\infty}^{\underline{\theta}} D(\theta) \frac{1}{\sigma_\eta} f_\eta \left(\frac{\theta_i - \theta}{\sigma_\eta}\right) d\theta$ is positive and strictly decreasing in θ_i, this means that there is a value of $\theta_i > -\infty$, for which $v\left(\theta_i, \tilde{\theta}\right) = 0$. We denote this value of θ_i by $\underline{\zeta}_1$. Therefore, if no other agents attack for any signal realization it is optimal to attack for any signal realizations below $\underline{\zeta}_1$. Next note that $v\left(\underline{\zeta}_n, \underline{\zeta}_n\right) > 0$ for any $\underline{\zeta}_n < \theta^*$, given the properties of the function $\tilde{v}\left(\tilde{\theta}\right)$, described above. Therefore, since $v\left(\theta_i, \tilde{\theta}\right)$ is strictly decreasing in θ_i, it follows that the sequence $\left\{\underline{\zeta}_n\right\}_{n=0}^{\infty}$ is strictly increasing and bounded above by θ^*. By continuity of v in both arguments, it follows that the sequence converges to $\underline{\zeta}_\infty = \theta^*$. Similarly, one can show that the sequence $\left\{\overline{\zeta}_n\right\}_{n=0}^{\infty}$ is strictly decreasing and bounded below by θ^*, so the sequence converges to $\overline{\zeta}_\infty = \theta^*$. Therefore, the monotone strategy with $a_i = 1$ for $\theta_i < \theta^*$ survives iterated deletion of strictly dominated strategies. Therefore, it is the (essentially) unique BNE of the game.

□

Proof of Proposition 2

Proof. Let $y = \frac{\theta - \theta^f}{\sigma_\eta} - F_\eta^{-1}\left(g\left(\theta^f\right)\right)$. Then, $\theta = \sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f$, $d\theta = \sigma_\eta dy$, and $\theta = \theta^f \implies y = -F_\eta^{-1}\left(g\left(\theta^f\right)\right)$. So, after this change of variables, we can write expression (10) as

$$\begin{aligned}\psi\left(\theta^f, \sigma_\eta\right) \equiv & \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} U\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right) f_\eta(-y)\, dy + \\ & \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} D\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right) f_\eta(-y)\, dy.\end{aligned} \tag{A.7}$$

Then,

$$\begin{aligned}\frac{\partial \psi}{\partial \theta^f} = & \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right] f_\eta\left(-F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) \frac{g_\theta\left(\theta^f\right)}{f_\eta\left(F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)} + \\ & + \left[1 + \sigma_\eta \frac{g_\theta\left(\theta^f\right)}{f_\eta\left(F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)}\right] \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} U'\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right) f_\eta(-y)\, dy \\ & + \left[1 + \sigma_\eta \frac{g_\theta\left(\theta^f\right)}{f_\eta\left(F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)}\right] \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} D'\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right) f_\eta(-y)\, dy < 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \psi}{\partial \sigma_\eta} = & \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} U'\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right)\left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy \\ & + \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} D'\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f\right)\left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy \\ = & \frac{1}{\sigma_\eta} \int_{\theta^f}^{+\infty} U'(\theta)\left(\theta - \theta^f\right) \frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) d\theta + \frac{1}{\sigma_\eta} \int_{-\infty}^{\theta^f} D'(\theta)\left(\theta - \theta^f\right) \frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) d\theta\end{aligned} \tag{A.8}$$

As a consequence, using the implicit function theorem, the sign of $\frac{\partial \theta^f}{\partial \sigma}|_{\psi=0}$ is simply the same as the sign of expression (A.8). This leads directly to condition (11).

□

Proof of Corollary 1

Proof. Suppose that $U(\theta) = U$, $\forall\theta$ and $D(\theta)$ is strictly decreasing. Then, $\frac{\partial\theta^f}{\partial\sigma_\eta} > 0$ since

$$\frac{E_{\theta^*}\left[\left|U'(\theta)\right|\left(\theta-\theta^f\right) \mid \theta > \theta^f\right]}{E_{\theta^*}\left[\left|U(\theta)\right| \mid \theta > \theta^f\right]} = 0 < \frac{E_{\theta^*}\left[\left|D'(\theta)\right|\left(\theta^f-\theta\right) \mid \theta \leq \theta^f\right]}{E_{\theta^*}\left[D(\theta) \mid \theta \leq \theta^f\right]}.$$

Suppose that $D(\theta) = D$, $\forall\theta$ and $U(\theta)$ is strictly decreasing, $\forall\theta$. Then from Proposition 2

$$\frac{E_{\theta^*}\left[\left|U'(\theta)\right|\left(\theta-\theta^f\right) \mid \theta > \theta^f\right]}{E_{\theta^*}\left[\left|U(\theta)\right| \mid \theta > \theta^f\right]} > \frac{E_{\theta^*}\left[\left|D'(\theta)\right|\left(\theta^f-\theta\right) \mid \theta \leq \theta^f\right]}{E_{\theta^*}\left[D(\theta) \mid \theta \leq \theta^f\right]} = 0$$

for any σ_η and θ^f and so $\frac{\partial\theta^f}{\partial\sigma_\eta} < 0$ for any σ_η and θ^f.

When $U'(\theta) = D'(\theta) = 0$, equation (A.8) and the implicit function theorem jointly imply that $\frac{\partial\theta^f}{\partial\sigma_\eta} = 0$.

□

Proof of Corollary 2

Proof. Consider Proposition 2 and equation (11): $E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta-\theta^f\right)\right] < 0$. We use a change of variables $y = \frac{\theta-\theta^f}{\sigma_\eta}$ to rewrite this condition as

$$\int_{-\infty}^{0}\left|D'\left(\theta^f+\sigma_\eta y\right)\right| y\sigma_\eta f_\eta\left(F_\eta^{-1}\left(g\left(\theta^f\right)\right)-y\right)dy+$$
$$+\int_{0}^{\infty}\left|U'\left(\theta^f+\sigma_\eta y\right)\right| y\sigma_\eta f_\eta\left(F_\eta^{-1}\left(g\left(\theta^f\right)\right)-y\right)dy < 0$$

Since η is symmetric, it follows that $F_\eta^{-1}\left(1-g\left(\theta^f\right)\right) = -F_\eta^{-1}\left(g\left(\theta^f\right)\right)$. Furthermore, since f_η is even, we can re-write the condition as

$$\int_{0}^{\infty}\left[\left|U'\left(\theta^f+\sigma_\eta y\right)\right| f_\eta\left(y+F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right)-\right.$$
$$\left.-\left|D'\left(\theta^f-\sigma_\eta y\right)\right| f_\eta\left(y-F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right)\right] y\sigma_\eta dy < 0$$

Therefore, if

$$\left|U'\left(\theta^f+\sigma_\eta y\right)\right| f_\eta\left(y+F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right) > \left|D'\left(\theta^f-\sigma_\eta y\right)\right| f_\eta\left(y-F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right), \forall y \geq 0, \quad \text{(A.9)}$$

then $E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta-\theta^f\right)\right] > 0$, and so $\frac{\partial\theta^f}{\partial\sigma_\eta} < 0$. Conversely, if

$$\left|U'\left(\theta^f+\sigma_\eta y\right)\right| f_\eta\left(y+F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right) < \left|D'\left(\theta^f-\sigma_\eta y\right)\right| f_\eta\left(y-F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right), \forall y \geq 0, \quad \text{(A.10)}$$

then $E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta-\theta^f\right)\right] < 0$, and so $\frac{\partial\theta^f}{\partial\sigma_\eta} > 0$.

Since $f_\eta(y)$ is decreasing in y for $y \geq 0$, it follows that $f_\eta\left(y+F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right) \geq f_\eta\left(y-F^{-1}\left(1-g\left(\theta^f\right)\right)\right)$, $\forall y \geq 0$ for $F_\eta^{-1}\left(1-g\left(\theta^f\right)\right) \leq 0$, or $g\left(\theta^f\right) \geq \frac{1}{2}$. Therefore, if $g\left(\theta^f\right) \geq \frac{1}{2}$ and $\left|U'\left(\theta^f+y\right)\right| > \left|D'\left(\theta^f-y\right)\right|$, $\forall y \geq 0$, then condition (A.9) holds, and so $\frac{\partial\theta^f}{\partial\sigma_\eta} < 0$. Thus, if π is dominated by upside risks at θ^f and $g\left(\theta^f\right) \geq \frac{1}{2}$, then $\frac{\partial\theta^f}{\partial\sigma_\eta} < 0$.

Similarly, using

$$f_\eta\left(y+F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right) \leq f_\eta\left(y-F_\eta^{-1}\left(1-g\left(\theta^f\right)\right)\right), \forall y \geq 0,$$

for $F_\eta^{-1}\left(1-g\left(\theta^f\right)\right) \geq 0$ or $g\left(\theta^f\right) \leq \frac{1}{2}$ we arrive at the second result that if $\left|U'\left(\theta^f+y\right)\right| < \left|D'\left(\theta^f-y\right)\right|$, $\forall y \geq 0$, and $g\left(\theta^f\right) \leq \frac{1}{2}$, then $\frac{\partial\theta^f}{\partial\sigma_\eta} > 0$.

□

Proof of Proposition 3

Proof. Assumption (A3) allows the exchange of limits and integrals, by using the dominated convergence theorem. Then, in the limit where noise disappears, expression (A.8) above can be written as

$$\begin{aligned} \frac{\partial \psi}{\partial \sigma_\eta} &= U'\left(\theta^f\right) \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy \\ &+ D'\left(\theta^f\right) \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy \end{aligned} \tag{A.11}$$

For any $\sigma_\eta > 0$, we have that $\frac{\theta^* - \theta}{\sigma_\eta} \sim F_\eta$. Let $x = \frac{\theta - \theta^*}{\sigma_\eta}$. We also have that away from the limit $\theta^*\left(\theta^f, \sigma_\eta\right) = \theta^f + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right)$.[19] Then, away from the limit it is always the case that

$$E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma_\eta} \mid \theta > \theta^f\right] \Pr_{\theta^*}\left[\theta > \theta^f\right] = \int_{\theta^f}^{+\infty} \frac{\theta - \theta^f}{\sigma_\eta} \frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) d\theta = \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy \tag{A.12}$$

So, we treat $\int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy$ as the limit of $E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma_\eta} \mid \theta > \theta^f\right] \Pr_{\theta^*}\left[\theta > \theta^f\right]$, as $\sigma_\eta \to 0$. An analogous result can be found for the other truncated integral.

Since $\lim_{\sigma_\eta \to 0} \Pr_{\theta^*}\left[\theta > \theta^f\right] = g\left(\theta^f\right)$, we can write that in the limit

$$\begin{aligned} \frac{\partial \psi}{\partial \sigma_\eta}\Big|_{\psi=0} &> 0 \\ &\Longleftrightarrow \\ U'\left(\theta^f\right) g\left(\theta^f\right) \lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma} \mid \theta > \theta^f\right] &> -D'\left(\theta^f\right)\left(1 - g\left(\theta^f\right)\right) \lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma} \mid \theta \leq \theta^f\right] \end{aligned} \tag{A.13}$$

Also, notice that $\lim_{\sigma \to 0} \psi_{\theta^f}$ exists and is positive, so $\lim_{\sigma_\eta \to 0} \frac{\partial \theta^f}{\partial \sigma_\eta}$ is well-defined. Additionally,

$$\begin{aligned} \lim_{\sigma_\eta \to 0} \psi\left(\theta^f, \sigma_\eta\right) &= \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} U\left(\theta^f\right) f_\eta(-y)\, dy + \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} D\left(\theta^f\right) f_\eta(y)\, dy \\ &= U\left(\theta^f\right) g\left(\theta^f\right) + D\left(\theta^f\right)\left(1 - g\left(\theta^f\right)\right) \end{aligned} \tag{A.14}$$

Finally, we define

$$\begin{aligned} h\left(g\left(\theta^f\right)\right) &\equiv - \frac{g\left(\theta^f\right)}{1 - g\left(\theta^f\right)} \frac{\int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy}{\int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} \left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy} \\ &= - \frac{\lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma} \mid \theta \leq \theta^f\right]}{\lim_{\sigma_\eta \to 0} E_{\theta^*}\left[\frac{\theta - \theta^f}{\sigma} \mid \theta > \theta^f\right]}. \end{aligned} \tag{A.15}$$

Combining (A.13), (A.14) and (A.15) we obtain the statement.

To show that $h(.)$ is strictly increasing if $\eta \sim N(0,1)$, we first solve the two integrals and rewrite $h(\cdot)$ as

$$h(g) = \frac{g}{1 - g} \frac{\phi\left(\Phi^{-1}(1 - g)\right) + \Phi^{-1}(1 - g)(1 - g)}{\phi\left(\Phi^{-1}(1 - g)\right) - \Phi^{-1}(1 - g)\, g} \tag{A.16}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and cumulative distributions of a standard normal random variable. We can

[19] Here $\theta^f(\sigma_\eta)$ is implicitly defined as the solution to $\psi\left(\theta^f, \sigma_\eta\right) = 0$. For brevity, we abuse notation and write simply θ^f.

simplify this expression further to:

$$h(g) = \frac{\frac{\phi\left(\Phi^{-1}(1-g)\right)}{\Phi^{-1}(1-g)}}{(1-g)\left[\frac{\phi\left(\Phi^{-1}(1-g)\right)}{\Phi^{-1}(1-g)} - g\right]} - 1 \tag{A.17}$$

The derivative of this function is given by

$$\frac{dh}{dg} = \frac{1}{\phi^2\left(\Phi^{-1}(1-g)\right)} \frac{\phi^2\left(\Phi^{-1}(1-g)\right) + (1-2g)\Phi^{-1}(1-g)\,\phi\left(\Phi^{-1}(1-g)\right) - g(1-g)\left(1+\left(\Phi^{-1}(1-g)\right)^2\right)}{(1-g)^2\left[\frac{\phi\left(\Phi^{-1}(1-g)\right)}{\Phi^{-1}(1-g)} - g\right]^2} \tag{A.18}$$

The numerator can be written as:

$$\left(\phi\left(\Phi^{-1}(1-g)\right) + (1-g)\Phi^{-1}(1-g)\right)\left(\phi\left(\Phi^{-1}(1-g)\right) - g\Phi^{-1}(1-g)\right) - g(1-g)$$

Using $\Phi^{-1}(g) = -\Phi^{-1}(1-g)$, we get the function:

$$\mu(g) = \left(\phi\left(\Phi^{-1}(g)\right) + g\Phi^{-1}(g)\right)\left(\phi\left(\Phi^{-1}(1-g)\right) + (1-g)\Phi^{-1}(1-g)\right) - g(1-g) \tag{A.19}$$

Note that $\mu(g) = \mu(1-g)$ and $\lim_{g\to 0}\mu(g) = 0$. Furthermore, one can show that $\mu(g)$ is strictly decreasing in g for $g \leq \frac{1}{2}$, so $\mu(g) < 0$ for $g \in (0,1)$. This implies that $\frac{dh}{dg} < 0$.

□

Proof of Proposition 4

The unique equilibrium is characterized by two conditions. The first one is an indifference condition for an agent that observes a signal $\theta = \theta^*$ given by

$$\hat{\psi}\left(\theta^f, \mu(\theta^*, \rho), \sigma\right) \equiv \int_{\theta^f}^{\infty} U(\theta)\,\phi\left(\frac{\theta - \mu(\theta^*, \rho)}{\sigma}\right)\frac{1}{\sigma}d\theta + \int_{-\infty}^{\theta^f} D(\theta)\,\phi\left(\frac{\theta - \mu(\theta^*, \rho)}{\sigma}\right)\frac{1}{\sigma}d\theta = 0.$$

where $\mu(\theta^*, \rho) = \rho^2\theta^* + \left(1-\rho^2\right)\theta_0$. The second is a condition relating θ^* and θ^f via the fraction of agents that attack the status quo, which is given by

$$\theta^* = \theta^f - \frac{\sigma}{\rho} v\left(\theta^f\right), \tag{A.20}$$

where $v\left(\theta^f\right) \equiv \Phi^{-1}\left(1 - g\left(\theta^f\right)\right)$ is a decreasing function. Combining both conditions, we obtain a single condition given by:

$$\hat{\psi}\left(\theta^f, \rho^2\left(\theta^f - \frac{\sigma}{\rho}v\left(\theta^f\right)\right) + \left(1-\rho^2\right)\theta_0, \sigma\right) = 0$$

We use this equilibrium condition and apply the Implicit Function Theorem. Note that

$$\frac{d\hat{\psi}}{d\theta^f} = \hat{\psi}_{\theta^f} + \hat{\psi}_{\mu}\left(\rho^2 - \sigma\rho v'\left(\theta^f\right)\right) \tag{A.21}$$

where

$$\hat{\psi}_{\theta^f} = -\left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\phi\left(\frac{\theta^f - \mu}{\sigma}\right)\frac{1}{\sigma} \tag{A.22}$$

and

$$\hat{\psi}_{\mu} = \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\frac{1}{\sigma}\phi\left(\frac{\theta^f - \mu}{\sigma}\right) - E_{\theta^*}\left[S\left(\theta, \theta^f\right)\right]$$

with $\mu = \rho^2 \left(\theta^f - \frac{\sigma}{\rho} v\left(\theta^f\right)\right) + \left(1-\rho^2\right)\theta_0$. Therefore,

$$\frac{d\hat{\psi}}{d\theta^f} = \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right] \frac{1}{\sigma}\phi\left(\frac{\theta^f - \mu}{\sigma}\right)\left(\rho^2 - 1 - \sigma\rho v'\left(\theta^f\right)\right) - E_{\theta^*}\left[S\left(\theta, \theta^f\right)\right]\left(\rho^2 - \sigma\rho v'\left(\theta^f\right)\right) \tag{A.23}$$

Also, we have that

$$\frac{\partial\hat{\psi}}{\partial\sigma} = \left[\left(1-\rho^2\right)\frac{\theta^f - \theta_0}{\sigma}\right]\hat{\psi}_\mu - \frac{1}{\sigma}E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta - \theta^f\right)\right] \tag{A.24}$$

and

$$\frac{\partial\hat{\psi}}{\partial\rho} = \frac{\partial\mu^*}{\partial\rho}\hat{\psi}_\mu = \left[2\rho\left(\frac{\theta^f + \theta^*}{2} - \theta_0\right)\right]\hat{\psi}_\mu \tag{A.25}$$

Clearly, in the limit as $\rho \to 1$, we have:

$$\lim_{\rho\to 1}\frac{d\hat{\psi}}{d\theta^f} = \left[\left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\frac{1}{\sigma}\phi\left(\frac{\theta^f - \mu}{\sigma}\right) - E_{\theta^*}\left[S\left(\theta,\theta^f\right)\right]\right]\left(1 - \sigma v'\left(\theta^f\right)\right) < 0 \tag{A.26}$$

since $v'\left(\theta^f\right) < 0$ given that $g_\theta > 0$. Therefore, the sign of $\frac{\partial\theta^f}{\partial\sigma}$ is the same as the sign of $\frac{d\hat{\psi}}{d\sigma}$, which in the limit as $\rho \to 1$ equals

$$-\frac{1}{\sigma}E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta - \theta^f\right)\right]$$

This gives our first result. Similarly, the sign of $\frac{\partial\theta^f}{\partial\rho}$ is the same as the sign of $\frac{d\hat{\psi}}{d\rho}$, which equals

$$\frac{d\hat{\psi}}{d\rho} = \left[2\rho\left(\frac{\theta^f + \theta^*}{2} - \theta_0\right)\right]\hat{\psi}_\mu \tag{A.27}$$

and so in the limit, as $\rho \to 1$,

$$\lim_{\rho\to 1}\frac{d\hat{\psi}}{d\rho} = \left[2\left(\frac{\theta^f + \theta^*}{2} - \theta_0\right)\right]\hat{\psi}_\mu \tag{A.28}$$

Since $\hat{\psi}_\mu < 0$, it follows that the sign of $\lim_{\rho\to 1}\frac{d\hat{\psi}}{d\rho}$ depends on the sign of $\theta_0 - \frac{\theta^f + \theta^*}{2}$, which gives our second result.

Proof of Proposition 5

Proof. The proof follows closely the proof of Proposition 2. We define

$$\Psi\left(\theta^f, \sigma_\eta\right) \equiv \int_{\theta^f}^{+\infty} U\left(\theta, F_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)\right)\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)d\theta + \int_{-\infty}^{\theta^f} D\left(\theta, F_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)\right)\frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)d\theta$$

After a change of variables ($y = \frac{\theta - \theta^*}{\sigma_\eta}$), we have that

$$\Psi\left(\theta^f, \sigma_\eta\right) = \int_{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)}^{+\infty} U\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f, F_\eta\left(-y\right)\right) f_\eta\left(-y\right)dy + \int_{-\infty}^{-F_\eta^{-1}\left(g\left(\theta^f\right)\right)} D\left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f, F_\eta\left(-y\right)\right) f_\eta\left(-y\right)dy \tag{A.29}$$

Implicit differentiation delivers: $\frac{\partial \psi}{\partial \theta^f} < 0$ and

$$\frac{\partial \psi}{\partial \sigma_\eta} = \int_{-F_\eta^{-1}(g(\theta^f))}^{+\infty} U_\theta \left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f, F_\eta(-y)\right)\left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy +$$
$$+ \int_{-\infty}^{-F_\eta^{-1}(g(\theta^f))} D_\theta \left(\sigma_\eta y + \sigma_\eta F_\eta^{-1}\left(g\left(\theta^f\right)\right) + \theta^f, F_\eta(-y)\right)\left(y + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right) f_\eta(-y)\, dy =$$
$$= \frac{1}{\sigma_\eta}\int_{\theta^f}^{+\infty} U_\theta\left(\theta, F_\eta\left(\frac{\theta^f - \theta}{\sigma_\eta} + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)\right)\left(\theta - \theta^f\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) d\theta +$$
$$+ \frac{1}{\sigma_\eta}\int_{-\infty}^{\theta^f} D_\theta\left(\theta, F_\eta\left(\frac{\theta^f - \theta}{\sigma_\eta} + F_\eta^{-1}\left(g\left(\theta^f\right)\right)\right)\right)\left(\theta - \theta^f\right)\frac{1}{\sigma_\eta} f_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) d\theta$$

Therefore, by the implicit function theorem, the sign of $\frac{\partial \theta^f}{\partial \sigma_\eta}$ is the same as the sign of $\frac{\partial \psi}{\partial \sigma_\eta}$, which gives rise to condition (19).

□

Proof of Corollary 3

Proof. In that model, $g(\theta) = \frac{1}{r}$, which implies that θ^f and θ^* satisfy the condition$F_\eta\left(\frac{\theta^* - \theta^f}{\sigma_\eta}\right) = g\left(\theta^f\right) = \frac{1}{r}$ and that the mass of runners $A(\theta, \theta^*) = F_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)$. The payoff differentials are given by $U(\theta, A) = r - \left(\frac{1 - Ar}{1 - A}\right) Rp(\theta)$ and $D(\theta, A) = \frac{1}{A}$. Therefore, $U_\theta(\theta, A(\theta, \theta^*)) = -\left(\frac{1 - rA(\theta, \theta^*)}{1 - A(\theta, \theta^*)}\right) Rp'(\theta) \leq 0$ for $\theta \geq \theta^f$ with equality for $\theta = \theta^f$ and $D_\theta\left(\theta, \Phi\left(\frac{\theta - \theta^*}{\sigma_\eta}\right)\right) = 0$. Then, it immediately follows that $E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] = -\int_{\theta^f}^{\infty} U_\theta(\theta, A(\theta, \theta^*))\, d\theta > 0$. Proposition 5 then implies that $\frac{\partial \theta^f}{\partial \sigma_\eta} < 0$.

□

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B Online Supplementary Material (not for publication)

Supplement to Section 4.1

Information quality effects away from the limit $\rho \to 1$

We examine the effect of absolute and relative information quality on regime change away from the limit $\rho \to 1$. We first consider the behavior of an agent who knows that the regime change occurs for $\theta \leq \theta^f$ and has posterior beliefs centered at some μ. This agent chooses $a_i = 1$, if and only if

$$\hat{\psi}\left(\theta^f, \mu, \sigma\right) \equiv \int_{\theta^f}^{\infty} U(\theta)\, \phi\left(\frac{\theta - \mu}{\sigma}\right) \frac{1}{\sigma} d\theta + \int_{-\infty}^{\theta^f} D(\theta)\, \phi\left(\frac{\theta - \mu}{\sigma}\right) \frac{1}{\sigma} d\theta \geq 0.$$

There are two important observations that can be made about $\hat{\psi}$. First, given our net payoff assumptions, we have

$$\hat{\psi}_\mu = \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right] \frac{1}{\sigma} \phi\left(\frac{\theta^f - \mu}{\sigma}\right) - E_{\theta_i}\left[S\left(\theta, \theta^f\right)\right] < 0$$

where $S\left(\theta, \theta^f\right) \equiv \begin{cases} \left|U'(\theta)\right| & , if\, \theta \geq \theta^f \\ \left|D'(\theta)\right| & , if\, \theta < \theta^f \end{cases}$. This is not surprising, since an increase in the posterior mean, μ, leads to a first-order stochastic dominance shift in the conditional belief distribution. Given that net payoffs are decreasing in θ, this implies that an agent's payoff from choosing $a_i = 1$ relative to $a_i = 0$ decreases.

Second, we can write

$$\hat{\psi}_\sigma = \frac{\left(\theta^f - \mu\right)}{\sigma} \hat{\psi}_\mu - \frac{1}{\sigma} E_{\theta_i}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] \tag{B.1}$$

An increase in the posterior variance of an agent's beliefs, holding the relative precision of public and private information (ρ) fixed, increases the dispersion of her belief around an unchanged mean. Expression (B.1) decomposes its consequences in two effects. The first term is exactly equivalent to a mean shift. As variance is increased, θ^f and μ become closer: they are fewer standard deviations apart. This term indicates that we can treat this effect as if the mean had changed, moving towards θ^f.[20] The second term is exactly the payoff sensitivity effect.

Taking θ^f as fixed, the root of $\hat{\psi}\left(\theta^f, \mu(\theta^*, \rho), \sigma\right) = 0$, in which $\mu(\theta^*, \rho) = \rho^2\theta^* + \left(1 - \rho^2\right)\theta_0$ defines implicitly the strategic cutoff θ^* as a function of θ^f. Also, given any strategic cutoff, θ^*, the

[20] Take, for instance, a situation in which $\theta^f > \mu$. An increase in variance makes the status quo a more likely event for an agent with beliefs centered around μ, as it increases the mass in the right tail of the distribution. As a consequence, it favors rollover ($a_i = 0$).

mass of agents that choose $a_i = 1$ when the state is θ is given by:

$$A(\theta) = \Phi\left(\frac{\theta^* - \theta}{\sigma_\eta}\right) = \Phi\left(\frac{\theta^* - \theta}{\frac{\sigma}{\rho}}\right).$$

This gives a second equation for the strategic cutoff:

$$\theta^* = \theta^f - \frac{\sigma}{\rho} v\left(\theta^f\right), \tag{B.2}$$

where $v\left(\theta^f\right) \equiv \Phi^{-1}\left(1 - g\left(\theta^f\right)\right)$ is a decreasing function. We can then write the mean belief for an agent at the equilibrium strategic cutoff θ^* as $\mu^*\left(\theta^f, \sigma, \rho\right) \equiv \rho^2\left(\theta^f - \frac{\sigma}{\rho} v\left(\theta^f\right)\right) + \left(1 - \rho^2\right)\theta_0$.

When we combine both equilibrium conditions, we obtain a single-equation equilibrium condition

$$\hat{\psi}\left(\theta^f, \rho^2\left(\theta^f - \frac{\sigma}{\rho} v\left(\theta^f\right)\right) + \left(1 - \rho^2\right)\theta_0, \sigma\right) = 0. \tag{B.3}$$

A change in absolute information quality, σ, induces two effects on it and, implicitly, on the failure cutoff. There is a direct effect, $\hat{\psi}_\sigma$, which as we discussed above consists of a "mean equivalent" response, $\frac{\left(\theta^f - \mu\right)}{\sigma}\hat{\psi}_\mu$, and a "payoff sensitivity" response, $\frac{1}{\sigma}E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right]$.

There is also an indirect effect through a change in the agent at the strategic cutoff, as equation (B.2) needs to be satisfied. This effect contributes with an additional $\frac{\partial\mu}{\partial\sigma}\hat{\psi}_\mu$. This arises because an increase in the posterior variance results in more dispersed signal realizations for agents away from the true value of the fundamentals when $\theta = \theta^f$. This increases the mass of agents with signals $\theta_i < \theta^*$, whenever $\theta^* < \theta^f$, and has the opposite effect when $\theta^* > \theta^f$. Since it takes $g\left(\theta^f\right)$ agents for regime change to occur at θ^f, this implies that the marginal agent needs to observe a more extreme signal, further away from θ^f.

Combining these effects, we get that

$$\frac{\partial\hat{\psi}}{\partial\sigma} = \left[\left(1 - \rho^2\right)\frac{\theta^f - \theta_0}{\sigma}\right]\hat{\psi}_\mu - \frac{1}{\sigma}E_{\theta^*}\left[S\left(\theta, \theta^f\right)\left(\theta - \theta^f\right)\right] \tag{B.4}$$

Unlike the effect of σ, a change in the relative quality of public versus private information, ρ, has only an effect through the posterior mean. In particular,

$$\frac{\partial\hat{\psi}}{\partial\rho} = \frac{\partial\mu^*}{\partial\rho}\hat{\psi}_\mu = \left[2\rho\left(\frac{\theta^f + \theta^*}{2} - \theta_0\right)\right]\hat{\psi}_\mu \tag{B.5}$$

We show the effects of σ and ρ on θ^f under the following additional condition.

A4 $\frac{d\hat{\psi}}{d\theta^f} < 0$.

It is well known (see, for example, Morris and Shin (2003)) that a low ρ can recover the multiplicity of equilibria, even in the presence of some private information. Condition A4 is sufficient for the absence of multiple solutions to equation B.3 and is always satisfied for ρ sufficiently close to 1. One can also re-write condition A4 as

$$\begin{aligned}\frac{d\hat{\psi}}{d\theta^f} &= \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\frac{1}{\sigma}\phi\left(\frac{\theta^f-\mu}{\sigma}\right)\left(\rho^2 - 1 - \sigma\rho v'\left(\theta^f\right)\right)\\ &- E_{\theta^*}\left[S\left(\theta,\theta^f\right)\right]\left(\rho^2 - \sigma\rho v'\left(\theta^f\right)\right)\end{aligned}$$

The second term is always negative while, the first term is negative for ρ sufficiently close to 1 given a value of θ^f (i.e. for any θ^f there is an $\overline{\rho} < 1$ s.t. for $\rho > \overline{\rho}$, $\rho^2 - 1 - \sigma\rho v'\left(\theta^f\right) > 0$. Therefore, one can equivalently state A4 based on model primitives.

Condition A4 also ensures that the comparative statics are well behaved. If condition A4 does not hold, all comparative statics change signs. This behavior corresponds to that around the "unstable" equilibrium in multiple equilibrium models.

Given this assumption, we characterize the comparative statics away from the limit $\rho \to 1$ in the following

Proposition 6. *Whenever (A4) is satisfied,*

$$\frac{\partial\theta^f}{\partial\sigma} > 0 \iff E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta - \theta^f\right)\right] < \left(1-\rho^2\right)\left(\theta^f - \theta_0\right)\hat{\psi}_\mu \tag{B.6}$$

and

$$\frac{\partial\theta^f}{\partial\rho} > 0 \iff \theta_0 > \frac{\theta^f + \theta^*}{2} \tag{B.7}$$

in which $\hat{\psi}_\mu < 0$.

Proof. We follow the steps in the proof of Proposition 4 in the paper. Note that

$$\begin{aligned}\frac{d\hat{\psi}}{d\theta^f} = \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\frac{1}{\sigma}\phi\left(\frac{\theta^f-\mu}{\sigma}\right)\left(\rho^2 - 1 - \sigma\rho v'\left(\theta^f\right)\right)\\ -E_{\theta^*}\left[S\left(\theta,\theta^f\right)\right]\left(\rho^2 - \sigma\rho v'\left(\theta^f\right)\right)\end{aligned} \tag{B.8}$$

Under condition A4, $\frac{d\hat{\psi}}{d\theta^f} < 0$. Therefore, the signs of $\frac{\partial\theta^f}{\partial\sigma}$ and $\frac{\partial\theta^f}{\partial\rho}$ follow directly from the derivatives $\frac{d\hat{\psi}}{d\sigma}$ and $\frac{d\hat{\psi}}{d\rho}$, which corresponds to the comparisons given in the proposition. □

Therefore, away from the limit $\rho \to 1$, the sign of $\frac{\partial\theta^f}{\partial\sigma}$ depends on a modified version of our main condition (11). In particular, since a change of σ has an additional effect that is equivalent to a change in the posterior mean, condition (B.6) compares the magnitude and direction of that effect against the magnitude and direction of the payoff sensitivity effect.

Information quality effects with "flat" payoff differentials

To contrast more clearly the effects arising from payoff sensitivities from effects arising through the relative position of the prior mean, θ_0, away from the limit $\rho \to 1$, it is instructive to consider a set of models, where net payoff differentials are not sensitive to changes in the state θ, a case that has been commonly studied in applied models featuring global games. In that case $E_{\theta^*}\left[S\left(\theta,\theta^f\right)\left(\theta-\theta^f\right)\right] = 0, \forall\theta^f$ and we have the following

Corollary 4. *Consider a regime change game with upside and downside payoff differentials that do not depend on θ. Whenever (A4) is satisfied,*

$$\frac{\partial\theta^f}{\partial\sigma} > 0 \iff \theta_0 > \theta^f \tag{B.9}$$

and

$$\frac{\partial\theta^f}{\partial\rho} > 0 \iff \theta_0 > \frac{\theta^f + \theta^*}{2}. \tag{B.10}$$

Proof. Follows directly from the proof of Proposition 6. □

Therefore, in a "flat payoffs" model, the effects of both absolute (σ) and relative (ρ) information quality depend only on the position of the prior mean relative to the regime change cutoff.

These implications are related to existing results in the global games literature on the interaction between information quality and the prior mean (Metz (2002), Bannier and Heinemann (2005), and Angeletos, Hellwig, and Pavan (2007)). In particular, Metz (2002) discusses this interaction in the context of a currency crisis model with flat payoffs. She finds that if the prior mean is sufficiently low, discounting it more by either putting more weight on private information (decreasing σ_η) or less weight on public information (increasing σ_ϵ), tends to decrease the regime change cutoff. Furthermore, Angeletos, Hellwig, and Pavan (2007) show that in the context of a dynamic regime change game with arrival of new private signals (and hence increasing precision of private information over time), the regime-change cutoff is decreasing in the precision of private information whenever the prior mean is sufficiently low and increasing in the precision of private information whenever the prior mean is sufficiently high.[21] Inspection of the effects of changes in ρ on θ^f summarized in Corollary 4, for the case when ρ is close to 1, so that any effects through σ are small, recovers these results.

Supplement to Section 4.3

Consider the problem of an agent prior to the realization of her idiosyncratic signal. Since agents are identical at this stage, it suffices to look at the expected utility of a representative agent.[22]

[21] See Theorem 1 in that paper and the discussion preceding it.

[22] The ex ante utility of a representative agent is a natural welfare measure used in the global games literature (see, for example, Angeletos and Pavan (2007) and Colombo, Femminis, and Pavan (2013))

Agents know σ_ϵ, σ_η and have rational expectations about the equilibrium value of θ^f. Each agent chooses the strategic cutoff, θ^*, to maximize expected utility.

In our previous positive analysis of equilibrium, it sufficed to describe net payoffs. This, however, is no longer true for welfare analysis. Therefore, let $\xi_{\mathcal{R}}(\theta)$ denote the payoff from action $a_i = 1$ when the realized regime is $\mathcal{R} \in \{0,1\}$. Then, given a realization θ for the fundamental, the probability that an agent will observe a signal $\theta_i > \theta^*$ and choose $a_i = 0$ is given by $1 - A(\theta, \theta^*, \sigma_\eta) = 1 - F_\eta\left(\frac{\theta^* - \theta}{\sigma_\eta}\right)$. This is also the mass of agents that choose $a_i = 0$, and as a consequence, the average payoff in the cross section of agents is

$$P\left(\theta, \theta^*, \theta^f, \sigma_\eta\right) \equiv \begin{cases} \xi_1(\theta) - \left(1 - A(\theta, \theta^*, \sigma_\eta)\right) D(\theta) & , if\, \theta < \theta^f \\ \xi_0(\theta) - \left(1 - A(\theta, \theta^*, \sigma_\eta)\right) U(\theta) & , if\, \theta \geq \theta^f. \end{cases} \tag{B.11}$$

The expected utility of the representative agent or *ex ante* welfare is then

$$\mathcal{W}\left(\theta^*, \theta^f, \sigma_\eta, \sigma_\epsilon\right) \equiv \int P\left(\theta, \theta^*, \theta^f, \sigma_\eta\right) \frac{1}{\sigma_\epsilon} f_\epsilon\left(\frac{\theta - \theta_0}{\sigma_\epsilon}\right) d\theta. \tag{B.12}$$

In equilibrium, θ^f and θ^* are related through

$$\zeta\left(\theta^*, \theta^f, \sigma_\eta\right) \equiv F_\eta\left(\frac{\theta^* - \theta^f}{\sigma_\eta}\right) - g\left(\theta^f\right) = 0. \tag{B.13}$$

When agents choose θ^*, they treat θ^f as fixed and do not internalize the effect of θ^* on θ^f arising through the equilibrium relationship (B.13). Therefore, agents choose θ^* to satisfy

$$\mathcal{W}_{\theta^*} = 0 \tag{B.14}$$

Condition (B.14) together with the equilibrium relationship (B.13) give the equilibrium use of information (Angeletos and Pavan (2007)) in the regime switching game - the strategic cutoff chosen in the Bayesian Nash Equilibrium of the game.

If a planner could mandate cutoff strategies, she would take into account the equilibrium relationship between θ^f and θ^* in order to evaluate

$$\frac{d\mathcal{W}}{d\theta^*} = \frac{\partial \mathcal{W}}{\partial \theta^*} + \frac{\partial \mathcal{W}}{\partial \theta^f} \frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0}. \tag{B.15}$$

The second term in this expression is an equilibrium externality which is not taken into account by individual agents: whenever they choose a lower θ^*, they exert a downward pressure on the regime switching cutoff θ^f and indirectly affect all other agents' welfare. In a natural constrained efficiency benchmark such as the one introduced by Angeletos and Pavan (2007), a planner chooses strategies which are not privately optimal, but does not transmit or aggregate any information. As a consequence, constrained efficiency requires

$$\frac{d\mathcal{W}}{d\theta^*} = 0. \tag{B.16}$$

Note that (B.16) provides a condition for the efficient use of information in the regime change game.

At the Bayesian Nash Equilibrium, the externality term is given by

$$\frac{\partial \mathcal{W}}{\partial \theta^f}\frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0} = \frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0}\left\{\left[\xi_1\left(\theta^f\right) - \xi_0\left(\theta^f\right)\right] + \left(1 - g\left(\theta^f\right)\right)\left[U\left(\theta^f\right) - D\left(\theta^f\right)\right]\right\}\frac{1}{\sigma_\epsilon}f_\epsilon\left(\frac{\theta^f - \theta_0}{\sigma_\epsilon}\right), \tag{B.17}$$

where $\frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0} > 0$. Its sign is, therefore, the same as the sign of the direct effect of θ^f on expected equilibrium welfare, $\frac{\partial \mathcal{W}}{\partial \theta^f}$. It is worth pointing out two features about this externality. First, generically, θ^f is at an inefficient level. From individual optimality of strategies, we have that $\frac{\partial \mathcal{W}}{\partial \theta^*} = 0$, so constrained efficiency requires $\frac{\partial \mathcal{W}}{\partial \theta^f}\frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0} = 0$, a condition which is generically false.[23] Second, whenever $\left[\xi_1\left(\theta^f\right) - \xi_0\left(\theta^f\right)\right] < 0$, so that the status quo regime ($\mathcal{R} = 0$) would weakly increase the utility even of agents that choose $a_i = 1$, as it holds in all our debt run examples, the externality from the choice of a higher strategic threshold is negative. In those cases, agents are too eager to run from a social perspective and too many financial institutions fail.

Consider a policy that affects the precision of public information. That policy has both a direct and an indirect effect on equilibrium welfare. In particular, under the equilibrium use of information,

$$\frac{d\mathcal{W}}{d\sigma_\eta} = \frac{\partial \mathcal{W}}{\partial \sigma_\eta} + \frac{\partial \theta^f}{\partial \sigma_\eta}\frac{\partial \mathcal{W}}{\partial \theta^f}\frac{\partial \theta^f}{\partial \theta^*}|_{\zeta=0} \tag{B.18}$$

where by an application of the envelope theorem, the effect of σ_η through θ^* is zero. The first term is the direct welfare effect of a decrease in private information precision, which is always negative, as the private value of information to agents is always positive. To see this formally, note that $\frac{\partial \mathcal{W}}{\partial \sigma_\eta} = \int \frac{\partial P}{\partial \sigma_\eta}\frac{1}{\sigma_\epsilon}f_\epsilon\left(\frac{\theta-\theta_0}{\sigma_\epsilon}\right)d\theta$, where $\frac{\partial P}{\partial \sigma_\eta} = \begin{cases} -\frac{\theta^*-\theta}{\sigma_\eta^2}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)U(\theta) & , \theta \geq \theta^f \\ -\frac{\theta^*-\theta}{\sigma_\eta^2}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)D(\theta) & , \theta < \theta^f \end{cases}$. Using $\frac{\partial \mathcal{W}}{\partial \theta^*} = \int \frac{\partial P}{\partial \theta^*}\frac{1}{\sigma_\epsilon}f_\epsilon\left(\frac{\theta-\theta_0}{\sigma_\epsilon}\right)d\theta = 0$, where $\frac{\partial P}{\partial \theta^*} = \begin{cases} \frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)U(\theta) & , \theta \geq \theta^f \\ \frac{1}{\sigma_\eta}f_\eta\left(\frac{\theta^*-\theta}{\sigma_\eta}\right)D(\theta) & , \theta < \theta^f \end{cases}$, one can show that $\frac{\partial \mathcal{W}}{\partial \sigma_\eta} = \frac{\partial \mathcal{W}}{\partial \sigma_\eta} - \frac{\theta^f-\theta^*}{\sigma_\eta}\frac{\partial \mathcal{W}}{\partial \theta^*} < 0$.

The second term is the indirect effect through the change in the externality. The sign of that effect depends on the sign of $\frac{\partial \theta^f}{\partial \sigma_\eta}$, which is the focus of our analysis in the paper.

[23] Given that the difference $\left[\xi_1\left(\theta^f\right) - \xi_0\left(\theta^f\right)\right]$ is strategically irrelevant, for any environment with a given Bayesian Nash Equilibrium, one can find a strategically equivalent environment with perturbed payoffs which has a non-zero externality and, therefore, features constrained inefficiency.

Supplement to Section 4.4

We assume that any change in an information acquisition parameter or a change in equilibrium beliefs occurs in the following way

$$\eta_i \sim H(\eta_i, \alpha) \equiv \alpha F_{\eta,1}(\eta_i) + (1-\alpha) F_{\eta,0}(\eta_i).$$

By studying small perturbations away from $F_{\eta,0}$, using the mixture formulation, we describe the consequences of a change in the distribution of private noise towards any arbitrary $F_{\eta,1}(\eta_i)$. With a diffuse prior, conditional posterior beliefs are given by

$$\theta \mid \theta_i \sim 1 - H(\theta_i - \theta, \alpha).$$

As in our previous analysis, there are two central loci to describe. Indifference of a marginal agent that received signal θ^* imposes

$$\psi\left(\theta^f, \theta^*, \alpha\right) \equiv \int_{\theta^f}^{\infty} U(\theta)\, h(\theta^* - \theta, \alpha)\, d\theta + \int_{-\infty}^{\theta^f} D(\theta)\, h(\theta^* - \theta, \alpha)\, d\theta. \tag{B.19}$$

Additionally, when agents follow a cutoff θ^*, the mass of agents choosing $a_i = 1$ for any realization θ of the state is $H(\theta^* - \theta, \alpha)$. Therefore, the second locus pins down the regime change cutoff as the root to

$$\chi\left(\theta^f, \theta^*, \alpha\right) \equiv H\left(\theta^* - \theta^f, \alpha\right) - g\left(\theta^f\right) = 0. \tag{B.20}$$

We focus on how changes in the information structure towards $F_{\eta,1}$ affect the regime-change cutoff.

Proposition 7. *As the private noise distribution is distorted away from $F_{\eta,0}(\eta_i)$, in the direction of $F_{\eta,1}(\eta_i)$, we have that*

$$\begin{aligned} \frac{\partial \theta^f}{\partial \alpha} > 0 \iff \quad & E_{F_{\eta,1}}\left[\hat{\pi}\left(\theta, \theta^f\right) \mid \theta^*\right] - E_{F_{\eta,0}}\left[\hat{\pi}\left(\theta, \theta^f\right) \mid \theta^*\right] > \\ & > \frac{\psi_{\theta^*}}{h(\theta^* - \theta^f, \alpha)}\left[F_{\eta,1}\left(\theta^* - \theta^f\right) - F_{\eta,0}\left(\theta^* - \theta^f\right)\right], \end{aligned} \tag{B.21}$$

in which $\hat{\pi}\left(\theta, \theta^f\right) \equiv \begin{cases} U(\theta) & , \text{if } \theta \geq \theta^f \\ D(\theta) & , \text{if } \theta < \theta^f \end{cases}$ *and where* $\frac{\psi_{\theta^*}}{h\left(\theta^* - \theta^f, \alpha\right)} < 0$.

Proof. From equation (B.20), we can describe locally the implicit function $\theta^*\left(\theta^f, \alpha\right)$. Then,

$$\frac{\partial \theta^f}{\partial \alpha} = -\frac{\psi_\alpha + \psi_{\theta^*} \frac{\partial \theta^*}{\partial \alpha}|_{\chi=0}}{\psi_{\theta^f} + \psi_{\theta^*} \frac{\partial \theta^*}{\partial \theta^f}|_{\chi=0}}.$$

We first show that the denominator is negative. Indeed,

$$\psi_{\theta^f} = -\left[U\left(\theta^f\right) - D\left(\theta^f\right)\right] h\left(\theta^* - \theta^f, \alpha\right) > 0,$$

$$\psi_{\theta^*} = \left[U\left(\theta^f\right) - D\left(\theta^f\right)\right] h\left(\theta^f - \theta^*\left(\theta^f, \alpha\right), \alpha\right) - E_H\left[S\left(\theta, \theta^f\right)\right] < 0,$$

and

$$\frac{\partial \theta^*}{\partial \theta^f}|_{\chi=0} = \frac{h\left(\theta^*\left(\theta^f, \alpha\right) - \theta^f, \alpha\right) + g'\left(\theta^f\right)}{h\left(\theta^*\left(\theta^f, \alpha\right) - \theta^f, \alpha\right)} > 1.$$

Therefore, $\psi_{\theta^f} + \psi_{\theta^*}\frac{\partial \theta^*}{\partial \theta^f}|_{\chi=0} < 0$

Regarding the numerator,

$$\psi_\alpha = E_{F_{\eta,1}}\left[\hat{\pi}\left(\theta, \theta^f\right) \mid \theta^*\right] - E_{F_{\eta,0}}\left[\hat{\pi}\left(\theta, \theta^f\right) \mid \theta^*\right]$$

and

$$\frac{\partial \theta^*}{\partial \alpha}|_{\chi=0} = -\frac{F_{\eta,1}\left(\theta^* - \theta^f\right) - F_{\eta,0}\left(\theta^* - \theta^f\right)}{h\left(\theta^*\left(\theta^f, \alpha\right) - \theta^f, \alpha\right)}$$

The proposition follows from these expressions. □

We interpret expression (B.21) in the following way. The terms on the left-hand side describe whether a change in the distribution from $F_{\eta,0}$ towards $F_{\eta,1}$ makes the marginal agent more inclined to choose $a_i = 1$. It takes the cutoff θ^f as given, so it does not take into account any equilibrium feedback effects. It represents a difference of expected net payoffs under the two distributions involved in the mixture. In particular, we can re-write it as

$$\begin{aligned}\left[D\left(\theta^f\right) - U\left(\theta^f\right)\right]\left[F_{\eta,0}\left(\theta^* - \theta^f\right) - F_{\eta,1}\left(\theta^* - \theta^f\right)\right] + \\ + \int_{-\infty}^{\infty} S\left(\theta, \theta^f\right)\left[F_{\eta,0}\left(\theta^* - \theta\right) - F_{\eta,1}\left(\theta^* - \theta\right)\right] d\theta\end{aligned} \tag{B.22}$$

with $S\left(\theta, \theta^f\right) = \begin{cases} \left|U'(\theta)\right| & , if\, \theta \geq \theta^f \\ \left|D'(\theta)\right| & , if\, \theta < \theta^f \end{cases}$. The first term reflects the jump in net payoffs at the regime change threshold (a form of a discrete sensitivity term). The two distributions lead to different probabilities of regime change from the point of view of an agent that observes a signal θ^* and the first term corresponds to the difference in expected payoffs as a result of this. The second term relates payoff sensitivities and the relative probability weights under the two distributions. Intuitively, a shift towards a belief distribution that puts more mass on sets where $D(\theta)$ is higher or, analogously, less mass where $|U(\theta)|$ is lower for a given signal θ^* induces agents to choose $a_i = 1$.

The right-hand side is related to equilibrium feedback effects between θ^* and θ^f. Notice that (B.20) imposes a positive relationship between the strategic and regime-change cutoffs. If agents are less likely to run for any θ_i, the regime-change cutoff θ^f decreases. When we move infinitesimally towards $F_{\eta,1}$, for fixed cutoffs, there is a change in the mass of agents receiving signals $\theta_i < \theta^*$ and choosing $a_i = 1$. This change is described by $\left[F_{\eta,1}\left(\theta^* - \theta^f\right) - F_{\eta,0}\left(\theta^* - \theta^f\right)\right]$. As a consequence, the equilibrium θ^* and, ultimately, θ^f must adjust in response to this change in α. This is the source of the second effect in condition (B.21).