Capital Budgeting and Risk Taking under Credit Constraints

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March 27, 2019

Abstract

Limited external financing creates a hedging motive that distorts resource allocation for investment projects. I study these distortions through a dynamic model with endogenous collateral constraints. The hedging motive can be broken into three components: expected future productivity, leverage capacity, and current net worth. While constrained firms behave as if averse to transitory fluctuations in net worth, they can endogenously pursue increased exposure to both persistent factors that predict future productivity and fluctuations in credit tightness. The most constrained firms abstain from financial hedging, while still distorting capital allocation decisions, thereby influencing firm-level volatility. These distortions contribute to a potential explanation for the negative cross-sectional relationship between volatility and net worth.

Keywords: capital budgeting, credit constraints, project selection, investment, risk exposure.
JEL codes: G31, G32, E22.

1 Introduction

Concerns about limited external financing affect both capital investment and financial planning. They introduce into these decisions a corporate hedging motive that distorts risk-taking. To study this distortion, I propose a dynamic model in which firms have access to multiple investment projects that differ in their risky revenues and their capacity to attract external finance. Operating a firm requires a dynamic plan for capital allocation, financing, and potential hedging policies. Endogenous collateral constraints, like in Rampini and Viswanathan (2010), limit both borrowing and financial hedging.

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I thank Heitor Almeida, António Antunes, Eduardo Azevedo, Carlos Carvalho, Leandro Gorno, Humberto Moreira, Adriano Rampini, Alp Simsek, and Vish Viswanathan; the department and associate editors; three anonymous referees; and seminar participants at BI, the Minneapolis Fed, FGV EESP, Insper, the 2013 Lubramacro, WB/CEMLA, SBE meetings, the 2014 E.S. Meeting of the Econometric Society, and the 2015 AFA meeting. I would also like to thank Abhijit Banerjee, Robert Townsend, and Iván Werning for their guidance in early stages of this project. Guido Maia provided excellent research assistance. Any remaining errors are my sole responsibility.
This study was financed, in part, by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES), Finance Code 001.

The main results are as follows. In using capital budgeting distortions for hedging, financially constrained firms depart from static profit maximization. Furthermore, attitudes toward risk (and the direction of hedging) are endogenous and depend on net worth, credit conditions, and the behavior of productivity shocks. In particular, the persistence of the productivity process plays a central role in shaping the direction of the hedging motive. Whenever shocks to the productivity process are sufficiently persistent, a capital allocation favoring projects with a higher exposure to these shocks increases the value of a financially constrained firm. This happens for empirically reasonable parameter values and helps rationalize a pattern of riskier low capitalization firms through endogenous risk-taking. I also study fluctuations in credit conditions and show that constrained firms, which are concerned about levered returns, might not have incentives to save resources to substitute for unreliable external finance.

Before detailing the mechanisms, I will illustrate the key trade-offs in this environment with an example of an airline evaluating expansion plans. While an investment in the expansion of a regional operation brings additional exposure to local economic conditions, an alternative attempt to establish a nationwide operation would diversify regional risks. In the absence of credit constraints, capital budgeting is standard: the firm should make all positive net present value investments and nothing else. In the presence of frictions, however, new trade-offs emerge: resources are likely insufficient for investment in all profitable routes, making direct comparisons necessary. On the one hand, a successful local expansion generates cash flows exactly when the business is prospering and reinvestment is particularly profitable. As such, it might help fund further expansion. On the other hand, diversification makes cash flows more stable, which is valuable when investment needs are difficult to predict. I analyze how the trade-off ultimately depends on the persistence of productivity.

Here, productivity should be understood to broadly encompass total factor productivity (TFP), input costs, demand fluctuations, and the combination of any additional factors that shape investment profitability.[1] Persistent shocks make firms value increased exposure. The origin of this effect lies in the need to partially self-finance. A persistent productivity improvement has two consequences. First, it increases cash flows, which can be used to fund investment and mitigate the effects of credit constraints. Second, it also increases profitability and the marginal value of investment.

Whenever a shock is sufficiently persistent, the second effect dominates. The firm's investment needs increase by more than cash flows can cover. Consequently, the firm finds itself relatively more constrained after positive productivity innovations. Conversely, when the innovation is negative, investment drops by more than cash flows contract and some downsizing can occur. When contemplating persistent shocks, firms face tighter constraints during the growth phase that follows a positive innovation than during a contraction phase that follows a negative one. An investment project that has a higher short-run exposure to factors determining the demand that a constrained firm faces, for example, becomes effectively more valuable to it, leading to investment distortions in its favor.[2] This occurs because a

[1] Rampini et al. (2014) study oil price insurance by airlines and explicitly show how input (and output) prices can be reduced to total factor productivity fluctuations.

[2] This dynamic effect does not originate from the convexity of the static profit function. It occurs while investment at date t is conditioned on the expected productivity at $t+1$, not on its ex post realization. The static convexity originates from the ability to condition factor demands and output levels on factor prices and productivity, whereas this dynamic effect originates from the dispersion in the marginal value of resources inside the firm across dates and states of the world.

project like this generates cash flows that are better aligned with future investment needs.[3]

I now return to the airline example: other decisions involve trade-offs between the operational and financial sides. One example is the selection of capital goods used. While fuel-efficient airplanes reduce operational costs, those particularly suitable for regional routes are many times difficult to redeploy and make for less-reliable forms of collateral.[4] A particular trade-off between cost efficiency and the stability in financing conditions emerges.

To assess general attitudes toward volatile external financial conditions, I study state-contingent credit constraints. Fluctuations in credit conditions create changes in achievable leverage and, for constrained firms, can lead to a (levered) marginal value of funds that moves positively with these fluctuations. In this case, firms lack incentives to secure internal funds for situations in which credit conditions deteriorate. Indeed, projects that have higher exposure to fluctuations in credit conditions would increase firm value: a more exposed project attracts more funds, especially when investment can be highly leveraged through additional collateralized borrowing. This result illustrates that, in the presence of fluctuations in borrowing capacity, the most constrained firms might fail to have incentives to use operational decisions to decrease exposure to these fluctuations. Actually, increasing underlying exposure can be value enhancing.

While the results are initially studied through two-period specializations of the model that allow for analytical solutions, an infinite-horizon setup is quantitatively evaluated in Section 4. I start with parameter values that are reasonable in view of the previous empirical studies and endow firms with projects that have different exposures to a single exogenous state that determines profitability. Constrained firms distort investment in favor of projects with a higher exposure to this underlying state and sacrifice short-run profits to facilitate growth under high profitability conditions.

In this setting, firms have multiple instruments for dealing with possible shocks, as they can defer dividend payments, accumulate net worth, engage in financial hedging, and also distort their investment decisions. I show that each instrument is used.[5] Even in the presence of many alternative instruments, distortions in capital budgeting are present. They are especially noticeable for smaller firms (i.e., those smaller than the median) and decrease for larger firms. These distortions also stabilize at small values for the top percentiles of firm size. Quantitatively, these distortions in capital allocation across projects are typically significantly smaller than the differences that would be induced by a 1% return differential across investment projects.

I additionally conduct an extensive sensitivity analysis. In particular, distortions increase with the persistence of shocks and the returns to scale parameter. Interestingly, whereas the hedging demand changes sign for a sufficiently small persistence of shocks, as expected from the theoretical analysis, it can

[3] This effect is reversed for transitory shocks, which have minor effects on future investment profitability but affect cash flows and the ability to finance operations in the short run. While constrained firms might fail to use financial instruments to hedge against transitory shocks, they are willing to distort capital budgeting decisions to reduce their exposure to them.

[4] See Benmelech and Bergman (2009, 2011) for empirical evidence of the impact of collateral quality on credit conditions and their volatility.

[5] In the foreign-exchange risk literature, Allayannis et al. (2001) and Bartram et al. (2010) document the coexistence of operational and financial risk management and quantify their relative roles. Bartram (2008) and Choi and Jiang (2009) raise concerns about the measurement of risk exposure, emphasizing the effect of operational strategies that transform high latent exposures into small measured fluctuations.

only lead to quantitatively negligible distortions, in that case. The intuition for this asymmetry between transitory and persistent shocks is that dividend postponement and financial hedging are sufficient instruments for dealing with temporary fluctuations, but are less effective against the long-lasting effects of persistent shocks. Also, the wider is the gap between (tax-adjusted) borrowing costs and lending rates, the larger capital budgeting distortions become. The intuition is the same as before: the more expensive potential substitutes for dealing with hedging demand become, the more the hedging motive manifests in capital budgeting.[6] Last, I allow collateral constraints and prices to respond to economic uncertainty and study their consequences. The key conclusion is that a countercyclical behavior associated with down payment requirements amplifies the capital budgeting distortions.

Relationship with the literature – For the modeling of financial contracts, I follow Rampini and Viswanathan (2010, 2013), who propose a set-up in which limited enforcement problems reduce to collateral constraints. This is similar to Kiyotaki and Moore (1997), but allows for state-contingent finance. The focus of those papers is on analyzing capital structure and risk management. Capital budgeting, like in most of the literature studying financial frictions, is reduced to the choice of the scale of investment in a single technology.[7] This work has successfully explained empirical regularities and previous puzzles in the cross-sectional profiles of risk management and leasing decisions.

The contribution relative to this literature is twofold. Most importantly, I analyze capital budgeting, focusing on the consequences of a hedging motive that is displayed even by firms that abstain from financial hedging. Like in previous work, the most constrained firms do not have slack left for financial hedging. Once capital budgeting is taken into account, these firms find that distorting capital investment decisions becomes a useful tool. Additionally, in empirically reasonable cases, distortions favor riskier projects.

As an intermediate step toward studying project selection, I review the determinants of the hedging motive. Although the risk management literature has emphasized the role of variations in net worth in shaping this motive, it has devoted less attention to state-contingent factors driving the marginal value of funds. I shed light on its dependence on both the expected productivity of a future investment, intrinsically related to the persistence of shocks, and on leverage possibilities, related to credit conditions.

The current paper is broadly related to a literature discussing allocative distortions and efficiency losses that originate from conflicts of interest in capital budgeting. One branch focuses on problems of delegation between owners and self-interested managers.[8] Another branch studies conflicts between equity and debt holders and discusses the risk-shifting problem that emerges when assets are not observable.[9] I abstract from both internal conflicts and imperfect observation of assets and complement

[6]Petersen and Thiagarajan (2000) and Kim et al. (2006) provide evidence of balancing the relative costs of alternative instruments.

[7]Consider, for instance, the canonical work of Froot et al. (1993) or that of Albuquerque and Hopenhayn (2004), Bolton et al. (2011), Clementi and Hopenhayn (2006), Holmström and Tirole (1998), and Krishnamurthy (2003).

[8]For instance, Harris and Raviv (1996), Harris and Raviv (1998), Rajan et al. (2000), and Stein (2002) study difficulties in allocating resources to a manager or multiple divisions with conflicting interests. Stein (2003) offers an excellent survey of early work. Szydlowski (Forthcoming) studies a dynamic moral hazard environment model in which a manager's efforts need to be allocated across multiple projects. The constrained-efficient allocation depends on a dynamic modified net present value rule, according to which risky projects are avoided if the firm is close to liquidation.

[9]Jensen and Meckling (1976) is the seminal reference of this literature. Landier et al. (2011) provide empirical evidence from the mortgage origination market.

this literature by studying how limited external finance alone shapes risk-taking incentives in capital budgeting.

Two other strands of literature offer macroeconomic perspectives. Stylized examples of project selection have appeared in work concerning the aggregate consequences of financial frictions.[10] I contribute to that strand by providing a more thorough analysis of the incentives in capital budgeting and risk management among financially constrained firms.[11] A second strand studies endogenous volatility and attempts to understand how trade-offs faced by firms help account for the empirical pattern of volatility across countries (Koren and Tenreyro, 2013) and along the business cycle (Decker et al., 2016). I add a cross-sectional dimension to that discussion by first illustrating that financially constrained firms have incentives to manipulate their risk exposure to facilitate self-financing.[12]

A closely related paper by Almeida et al. (2011) studies capital budgeting distortions induced by costly access to external funds. The authors describe choices made across a few particular projects that differ in liquidity and riskiness. They make a key assumption that projects are uncorrelated. A consequence of this assumption is that more constrained firms should increase both financial and operational hedging and thus end up with lower volatility. When I consider a more general structure for the relationship between current and future investment opportunities, the sign of hedging demand depends on the persistence of shocks. I show that calibrations relatively standard in the literature generate a hedging motive that makes smaller, constrained firms take up larger risk exposure, helping account for the empirical cross-sectional pattern.

Perhaps surprisingly, little empirical work documents and tests distortions in capital budgeting. One important exception is Krüger et al. (2015), whose starting point is the weighted-average cost of capital (WACC) fallacy: the use of a rule-of-thumb behavior in organizations to use a single WACC estimate to discount cash flows regardless of their heterogeneous risk exposures. Doing so makes excessive investment flow to risky divisions. They present evidence of this overinvestment. The empirical predictions from this paper are similar, but are due to a different rationale: when productivity innovations are sufficiently persistent, higher investment in riskier technologies is a justifiable hedging response.

Organization – The remainder of the paper is organized as follows. Section 2 proposes the baseline model of capital budgeting, financing, and risk management and provides an analysis of the key endogenous variable behind firm's decisions: the marginal value of internal funds. Sections 2.2 and 2.3 focus on distortions in the allocation of capital budgets. These general results are then specialized in Section 3 by imposing more structure on elements of the model to illustrate how firms evaluate expo-

[10]For instance, Aghion et al. (2010), Greenwood and Jovanovic (1990), and Matsuyama (2007, 2008). Eisfeldt and Rampini (2007) discuss how credit constraints affect the composition of investment across used and new vintages of capital and provide empirical evidence.

[11]This paper also creates a dialogue with the macroeconomic literature on misallocation by showing that, to facilitate their dynamic financing, constrained firms distort investment. These distortions should arise in measures of misallocation, in the form of additional dispersion of marginal product of capital, even within a single firm. Some of this literature has also studied the relationship between the persistence of shocks and the need to self-finance, without any particular focus on capital budgeting (see, for instance, Banerjee and Moll, 2010; Buera and Shin, 2011; Moll, 2014).

[12]Consider the evidence for cross-country comparisons in Koren and Tenreyro (2007), who show that firms in less-developed countries concentrate on more volatile sectors, and in Decker et al. (2016), who draw a comparison of mean volatility across the large-firm Compustat database versus the smaller-firm Kauffman Firm Survey. Similarly, Davis et al. (2007) draw a correlation between measures of growth volatility and typical proxies of financial constraints, such as size, age, and publicly traded status.

sure to productivity shocks with different degrees of persistence and to credit capacity shocks. Section 4 provides quantitative evaluations in a steady state and a thorough sensitivity analysis and the last section concludes. An online appendix includes an empirical motivation and two extensions, one dealing with infrequently changed decisions in capital budgeting and a second one studying additional frictions in financial hedging.

2 Model

I start by introducing a model of a firm's financial decisions that takes the set of available investment projects as given. This initial setup extends the results from Rampini and Viswanathan (2010), a risk management model in which state-contingent borrowing is limited by endogenous collateral constraints to incorporate capital budgeting decisions. I use this baseline model to analyze how limited credit, productivity innovations, and leverage possibilities shape the marginal value of the firm's internal funds across states of the world. This is the key variable driving distortions in the corporate assessment of risky projects.

Guided by that discussion, I then analyze project selection. Section 2.2 provides some general results on how potentially constrained firms assess investment in risky projects. Section 3 specializes these results.

The benchmark setup is as follows. Time is discrete and indexed by $t = 0, 1, ..., T$, with $T \leq +\infty$. Uncertainty is described by an exogenous event tree. The initial state s^0 is a singleton, and $s^t \in S^t$ denotes the history known at time t. I define the transition probabilities between node s^t and its successors s^{t+1}, $\pi\left(s^{t+1}|s^t\right)$, in the usual way and let $\pi\left(s^t\right)$ denote the unconditional probability of state $s^t \in S^t$ being reached.

The economy is populated by two types of risk-neutral agents. One, which I call *firms*, has access to production technologies. The other group comprises *lenders* who, without direct access to a production technology, provide external funding to firms.

A firm maximizes the expected discounted dividend stream according to

$$E\left[\sum_{t=0}^{T} \beta^t d_t\right],$$

where $\beta \leq 1$.

Different investment projects entail different exposures of cash flows to the most relevant risk factors, such as input and output prices as well as both idiosyncratic and aggregate productivity shocks. Projects might also differ in other relevant ways, such as by involving capital goods that can be more or less easily redeployed in alternative uses, serve as better collateral, or have different exposures to price fluctuations. In the airline example from the introduction, these were embedded in the decisions of which routes to explore and which aircraft to choose.

I represent a project type by $j \in \mathcal{J} = \{0, 1, ..., J\}$. Project selection is observable to lenders and can be contracted on. For concreteness, I allow projects to differ along these three dimensions: how much output is generated in each contingency given previous investment, the price of the capital goods used by the project, and the project's recovery rate.[13] One can think of the first as the exposure of cash flows to risks, the second as the fluctuations in the relevant cost of investment/divestment, and the third as sensitivity to variations in credit conditions. Thus, the project type determines the evolution of the production function, capital prices, and credit constraints as functions of the exogenous uncertainty embedded in s^t.

Formally, a firm running a given project type $j \in \mathcal{J}$ uses a type-specific capital good, which is traded at a price $q^j\left(s^t\right)$. Capital k_{t+1}^j of this type, purchased and installed in state s^t, generates revenues $F^j\left(k_{t+1}^j, s^{t+1}\right)$ and $(1-\delta)$ units of depreciated s^{t+1} capital of the same type. Here, $F^j\left(\cdot, s^{t+1}\right)$ is a concave differentiable production function.

Lenders have a discount factor of $R^{-1} \geq \beta$, have deep pockets, and are not subject to commitment problems, so they are willing to buy and sell contingent claims at an expected rate of return of R.[14] Markets are complete in the sense that assets based on all contingencies can be traded, that is, a full spanning notion. However, the firm's ability to issue claims on its output is limited by commitment problems.

At date t, after production takes place, a firm can renege on any of its outstanding debt. If that happens, lenders can only recoup a fraction, $\theta^j\left(s^t\right) \in [0,1]$, of the capital goods of type j. I refer to $\theta^j\left(s^t\right)$ as the *recovery rate*. Therefore, given a level of capital goods $\left\{k_t^j\right\}_{j\in\mathcal{J}}$ used across the different projects, lenders can, at most, expect to recover[15] a total of

$$\sum_{j\in\mathcal{J}} \theta^j\left(s^t\right) q^j\left(s^t\right)(1-\delta)\, k_t^j. \tag{1}$$

After reneging on its debt, the firm can go back to the capital markets with a net worth equal to all the cash flows it absconded with, plus the fraction $\left(1-\theta^j\left(s^t\right)\right)$ of the depreciated stock of each capital good j.

An extension of Rampini and Viswanathan (2010) shows that, in this setting, the enforcement constraints to be imposed on the firm's problem greatly simplify: the value of the outstanding level of debt in any state, $b\left(s^t\right)$, cannot exceed how much a lender would recover upon default, given in Equation (1). Additionally, without imposing any restrictions on the maturity structure of repayments, the financial contract can be implemented with a sequence of state-contingent short-term debt issues.

I use these results and, to streamline the presentation, define two key auxiliary variables. The first variable is the *down payment* required per unit of capital good of type j at node s^t,

$$\wp^j\left(s^t\right) \equiv q^j\left(s^t\right) - E\left[R^{-1}\theta^j\left(s^{t+1}\right)(1-\delta)\, q^j\left(s^{t+1}\right)|s^t\right]. \tag{2}$$

[13] Heterogeneous or stochastic depreciation rates can be accommodated with some additional notation.

[14] Rampini and Viswanathan (2013) and Li et al. (2016) justify the reasonableness of a strict inequality in $R < \beta^{-1}$ with the tax-base reduction benefit of most debt instruments. A strict inequality additionally ensures that an unconstrained firm, once subject to a sufficiently long sequence of negative shocks, eventually finds itself constrained again.

[15] Allowing for recovery of a fraction of output would not lead to any major departure from the results presented later.

The second variable is *financial slack*, or unused borrowing capacity,

$$h\left(s^{t+1}\right) \equiv \sum_{j\in\mathcal{J}} \theta^j\left(s^{t+1}\right)(1-\delta)\, q^j\left(s^{t+1}\right) k^j_{t+1} - b\left(s^{t+1}\right). \tag{3}$$

The down payment requirement is taken as given by the firm. It is defined as the minimum payment needed to deploy a unit of capital, that is, how much the firm spends up front when financing a purchase at a unit price $q^j\left(s^t\right)$ by borrowing all that lenders are willing to lend against that collateral. Financial slack (in Equation (3)) is a non-negative choice variable that represents the difference between the collateral value of the firm's capital stock in state s^{t+1}, that is, the borrowing capacity of the firm against that state, and how much the firm is actually pledging to repay from that state onward. That is, a firm that borrows less than the maximum possible is said to be conserving financial slack. Conservation of financial slack and state-contingent savings are indistinguishable in the recursive formulation that follows.

Given the absence of adjustment costs, capital can be reallocated across projects after production takes place. As a consequence, a single state variable, tracking the firm's net worth, suffices for this recursive formulation.

$$V\left(w, s^t\right) = \max_{\left\{d,\{k^j\}_j,\{h(s^{t+1})\}_{s^{t+1}|s^t}\right\}\geq 0} d + \beta E_t\left[V\left(w\left(s^{t+1}\right), s^{t+1}\right)\right] \tag{4}$$

s.t.

$$w \geq d + E\left[R^{-1} h\left(s^{t+1}\right)\right] + \sum_j \wp^j\left(s^t\right) k^j, \tag{5}$$

and

$$w\left(s^{t+1}\right) = \sum_{j\in\mathcal{J}} \left\{F^j\left(k^j, s^{t+1}\right) + \left(1-\theta^j\left(s^{t+1}\right)\right) q^j\left(s^{t+1}\right)(1-\delta)\, k^j\right\} + h^j\left(s^{t+1}\right). \tag{6}$$

The envelope theorem ensures that the multiplier on the first constraint equals the firm's *shadow value of net worth*, $\frac{\partial V\left(w,s^t\right)}{\partial w}$.[16] I will also refer to this as *marginal value of internal funds* interchangeably.

The solution to the recursive maximization problem is characterized by the following set of first-order conditions:

k^j :

$$\beta E_t\left[\frac{\partial V\left(w\left(s^{t+1}\right), s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}\left(\frac{\partial F^j\left(k^j, s^{t+1}\right)}{\partial k^j} + \left(1-\theta^j\left(s^{t+1}\right)\right)(1-\delta)\, q^j\left(s^{t+1}\right)\right)\right] \leq \wp^j\left(s^t\right)\frac{\partial V\left(w, s^t\right)}{\partial w}, \tag{7}$$

$$d:\ 1 \leq \frac{\partial V\left(w, s^t\right)}{\partial w}, \tag{8}$$

[16] As financial slack linearly enters the constraints, an interior solution is typically not guaranteed, and Benveniste and Scheinkman (1979) does not apply without making adaptations. One possibility for proving differentiability and the envelope result lies in using corollary 5.2 from Milgrom and Segal (2002) together with the uniqueness of the Lagrangian multipliers from 4, which can be established from the first-order conditions associated with problem 4.

and

$$h\left(s^{t+1}\right):\ \beta R\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}\leq\frac{\partial V\left(w,s^{t}\right)}{\partial w}, \tag{9}$$

each of which holds as an equality if the relevant choice variable is strictly positive.

Equation (7) represents the firm's capital investment Euler equation for project $j\in\mathcal{J}$. Guided by it, I define the *leveraged marginal return* on investment in project j as

$$R_{lev}^{j}\left(k^{j},s^{t+1}\right)\equiv\frac{\frac{\partial F^{j}\left(k^{j},s^{t+1}\right)}{\partial k^{j}}+\left(1-\theta^{j}\left(s^{t+1}\right)\right)\left(1-\delta\right)q^{j}\left(s^{t+1}\right)}{\wp^{j}\left(s^{t}\right)}.$$

This represents the variation in net worth induced by a marginal investment in capital good j associated with the maximum amount of borrowing possible against its collateral value. Therefore, I rewrite the investment Euler equation as

$$\frac{\partial V\left(w,s^{t}\right)}{\partial w}\geq\beta E_{t}\left[\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}R_{lev}^{j}\left(k^{j},s^{t+1}\right)\right], \tag{10}$$

which holds as an equality whenever the operation of that project takes place.

The capital accumulation equation indicates the importance of two intrinsically related endogenous variables: the value of internal funds and the marginal levered return. Their behavior is key for understanding how credit constraints influence the decisions of constrained firms, not only in terms of financial planning but also in terms of their real investment decisions. For that reason, I first take a deeper look at these variables.

2.1 The value of net worth

Standard dynamic programming arguments establish that the value function, $V\left(w,s^{t}\right)$, is concave in w, so that the marginal value of net worth is decreasing.[17] Additionally, when this concavity is strict, the marginal value reaches one for a sufficiently high net worth.

The concavity of the value function has been pointed out as a reason for risk management, along the argument first put forward by Froot et al. (1993): financially constrained firms become averse to fluctuations in net worth, because these fluctuations create an inefficient dispersion in the marginal value of investment across states of the world. However, state dependence is also part of the firm's problem. Besides net worth, leverage possibilities and expected productivity play central roles, as I will now concisely illustrate.

First, I impose Inada conditions for project $j=0$ to ensure its operation in all states. Let τ denote the random variable that describes the first dividend payment after time t, which is an endogenous decision. Then, at any such event s^{τ},

$$\frac{\partial V\left(w,s^{\tau}\right)}{\partial w}=1,$$

[17] Proposition 1 in Rampini and Viswanathan (2013) extends to the current setup.

and iterating backward toward s^t, one obtains

$$\frac{\partial V\left(w,s^t\right)}{\partial w}=E_t\left[\beta^{\tau-t}\prod_{h=t+1}^{\tau}R^0_{lev}\left(k^0\left(s^h\right),s^h\right)\right], \tag{11}$$

where $k^0\left(s^h\right)$ stands for the level of capital chosen at s^{h-1}.

From Equation (11), the marginal value of internal funds can be computed as an expected product of levered returns up to the next dividend payment.[18] All else equal, the more productive, the more levered , or the worse capitalized a firm is, the higher these returns are.

As a sample case to build intuition, let the set of projects $\mathcal{J}$ be a singleton. I temporarily drop the dependence of the notation on the projects. Let time be finite and indexed by three dates, $t\in\{0,1,2\}$; let the production function be separable as $F\left(k_{t+1},s^{t+1}\right)=A\left(s^{t+1}\right)k^{\alpha}_{t+1}$, with $\alpha\in(0,1)$; and let capital be fully pledgeable, like in Kiyotaki and Moore (1997), so that $\theta\left(s^t\right)=1$, for all t,s^t. I focus on $t=1$, one period before liquidation dividends are paid out.

Whenever $\frac{\partial V\left(w,s^1\right)}{\partial w}>1$, the firm is effectively constrained in its capital deployment decisions and uses maximum leverage to invest all its net worth in capital by purchasing $k_2\left(s^1\right)=\frac{w\left(s^1\right)}{\wp\left(s^1\right)}$. In that case, the marginal value of internal funds is

$$\frac{\partial V\left(w,s^1\right)}{\partial w}=\beta\alpha\frac{E\left[A\left(s^2\right)|s^1\right]}{\wp\left(s^1\right)}k_2\left(s^1\right)^{\alpha-1}=\beta\alpha\frac{E\left[A\left(s^2\right)|s^1\right]}{\wp\left(s^1\right)^{\alpha}}w\left(s^1\right)^{\alpha-1}. \tag{12}$$

Three effects are in place. The expected productivity effect, embedded in the $E\left[A\left(s^2\right)|s^1\right]$ term, pushes resources toward being more valuable in higher productivity states. The leverage effect, embedded in the reciprocal of the down payment requirement, increases the value of resources when the credit conditions are looser and the down payment is lower. Notice that decreasing returns to scale dampen this effect, but do not change its sign. Finally, the induced risk aversion effect, which is the effect most emphasized in the risk management literature, originates from the concavity of the production function and ensures that, *ceteris paribus*, firms with lower net worth face more severe distortions, deploy less capital, and have higher marginal returns to investment.

An interesting consumption-based asset pricing analogy can be drawn with the three effects behind Equation (12). Merton (1969, 1971) solved a continuous-time portfolio problem in the presence of underlying states that follow a diffusion. That solution is explicit up to the determination of the value function (that admits a closed-form in particular cases) and decomposes the demand for risky assets into two components.

One is a *myopic demand*, in which the individuals choose their exposure to risky assets as a function of return differentials. The propensity to increase exposure to assets with higher returns is disciplined by the variance in their returns weighted by the endogenous concavity of the value function. In this setup, the concavity that disciplines the demand for risky investment in projects manifests in $\frac{\partial^2 V}{\partial w_t^2}\left(w\left(s^1\right),s^1\right)\leq 0$, which follows from the differentiation of Equation (12).

[18] Equation (11) provides a useful measure of financial constraints and a moment condition that can be evaluated with data.

The second component in the demand for risky assets is what Merton called *hedging demand.* In a saver's portfolio problem, the hedging demand emerges from the interaction of income and substitution effects that make $\frac{\partial V\left(w,s^1\right)}{\partial w}$ state dependent.[19] In Equation (12), an analogous state dependence manifests through the influence of future expected productivity and leverage possibilities. If both are increasing (decreasing) in the underlying state, so is the marginal value of funds, and a hedging demand will incentivize firms to increase (decrease) their exposure to that state.

Next, I consider how these three effects interact when firms evaluate alternative investment projects and how they shape optimal financial policies. Unfortunately, unlike in Merton's problem, here an explicit solution cannot be obtained because of the presence of non-negativity constraints and decreasing returns to scale. A few additional lessons about the risk attitudes of constrained firms and optimal distortions in their capital budgeting processes emerge nonetheless. For example, Sections 3.1 and 4.1 discuss the importance of persistence in productivity processes in shaping a hedging demand, and Sections 3.2 and 4.2 deal with leverage and credit conditions. The following subsection takes the marginal value of funds as given and discusses the induced distortions in resource allocation.

2.2 Evaluating projects

Suppose the firm invests a positive amount in project $j = 0$ in its optimal investment plan. Let that firm evaluate the relative gains from allocating additional resources to some project $j = 1$ and, in doing so, moving them from $j = 0$. From the capital investment Euler equations (Equation 10) for these two projects, it follows that

$$E_t\left[\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}\left(R^1_{lev}\left(k^1,s^{t+1}\right)-R^0_{lev}\left(k^0,s^{t+1}\right)\right)\right]\leq 0, \tag{13}$$

with equality if investment occurs in both projects at the optimum. Whenever the inequality is strict when evaluated at $k^1 = 0$, the firm should refrain from investing anything in project $j = 1$.

As is standard in the asset pricing literature, I proceed toward rewriting that condition in a covariance form. To simplify the notation, I first define a return differential and a normalized marginal value of funds.

Let $\Delta R\left(k^1,s^{t+1}\right)\equiv R^1_{lev}\left(k^1,s^{t+1}\right)-R^0_{lev}\left(k^0,s^{t+1}\right)$ denote the *excess levered return* between these two projects as a function of investment in project $j = 1$, and let

$$m\left(s^{t+1}\right)\equiv\frac{\partial V\left(w,s^{t+1}\right)/\partial w}{E_t\left[\partial V\left(w,s^{t+1}\right)/\partial w\right]}$$

denote the normalized *marginal value of funds* at $t+1$ at the optimum.

Manipulating expression (13) leads to a decision rule that implies a decision to not operate project $j = 1$ whenever

[19] In Merton's problem, with a diffusion for s_t, the cross partial $\frac{\partial^2 V\left(w,s^t\right)}{\partial w_t \partial s_t}$ is well defined. Here, one needs to evaluate the discrete changes in $\frac{\partial V\left(w,s^t\right)}{\partial w}$ across states.

$$E_t\left[R^1_{lev}\left(0,s^{t+1}\right)\right]+Cov_t\left(m\left(s^{t+1}\right),\Delta R\left(0,s^{t+1}\right)\right)<E_t\left[R^0_{lev}\left(k^0,s^{t+1}\right)\right]. \tag{14}$$

Here, a few features grab attention. First, given that firms cannot borrow arbitrary amounts, project selection is always comparative: at the margin, any two projects compete for internal funds and become mutually exclusive. Under decreasing returns to scale, firms that are more constrained have higher leveraged marginal returns and, consequently, face naturally higher hurdle rates.

Second, the relevant return taken into account is a leveraged return, not a simple return on investment. A project capable of raising more collateralized financing allows for a lower down payment and, as a consequence, requires fewer resources to be displaced from other profitable opportunities the firm might have.

Third, constrained firms take into account a covariance term: projects that pay out more in the states in which the value of internal resources is higher are preferred. A lower-return project might be preferred over a higher-return project if it pays out more in the states in which the firm is most constrained. Results in Section 3.1 illustrate that when productivity is sufficiently persistent, firms are actually more constrained after positive, rather than negative, productivity innovations. As a consequence, Equation (14) indicates a positive covariance between $\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}$ and $R^0_{lev}\left(k^0,s^{t+1}\right)$. It follows that diversifying from the baseline project lowers the firm's value, even if the alternative project offers moderately higher expected returns.

Notice also that even in the absence of any technological interactions, such as economies of scope, frictions in access to external funding are capable of generating both substitution and complementarity across projects. Substitution is present when two contemporaneous projects that cannot be fully externally financed compete for the use of the firm's resources. A complementarity arises across time, because projects that offer payouts that positively covary with the marginal value of net worth help finance the firm's most productive investment opportunities. Therefore, although the firm is always maximizing the total net present value (NPV) of dividends by construction, it is not maximizing NPV project by project. A project is evaluated in light of its capital requirements, its ability to attract external funding, and its ability to generate additional funding for the most valuable investment opportunities.

Additionally, the standard net present value criterion can be recovered as a particular case. If I make the discount factor of lenders and firms the same by setting $\beta=R^{-1}$ and look at firms that are effectively unconstrained and pay out dividends at s^t, these firms face $\frac{\partial V\left(w,s^t\right)}{\partial w}=1$. It follows that $\frac{\partial V\left(w\left(s^{t+j}\right),s^{t+j}\right)}{\partial w\left(s^{t+j}\right)}=1$ holds for any node s^{t+j} that is a successor of s^t. Then Equation (14) collapses into the first-best rule of optimal investment: a firm should undertake a project if and only if it has a positive net present value.

2.3 Myers' adjusted present value criterion

In a seminal contribution, Myers (1974) was the first to study the interactions between financing and investment decisions in the presence of frictions. The paper analyzes a once-and-for-all investment decision in the presence of two exogenous functions that introduce the consequences of frictions in a reduced-form manner: a debt capacity function, which describes how much a firm could borrow as a function of investment in the different projects, and a value of cash inflows, which allows the firm's value to increase more than one-to-one with each dollar of cash flow generated.

The first-order conditions for capital investment in the different projects can be interpreted in light of Myers's seminal adjusted present value (APV) formulas. My main contribution in this dimension lies in incorporating a recursive stochastic structure that is used to describe the fundamentals behind objects that show up in Myers (1974) as shadow values and reduced-form functions.

First, I rewrite the capital investment Euler equation in a given project j (Equation 7) as

$$\beta E_t\left[\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}\left(\frac{\partial F^j\left(k^j,s^{t+1}\right)}{\partial k^j}+q^j\left(s^{t+1}\right)(1-\delta)\right)\right]$$
$$+E\left[\left(R^{-1}\frac{\partial V\left(w,s^t\right)}{\partial w}-\beta\frac{\partial V\left(w\left(s^{t+1}\right),s^{t+1}\right)}{\partial w\left(s^{t+1}\right)}\right)\theta^j\left(s^{t+1}\right)(1-\delta)\,q^j\left(s^{t+1}\right)|s^t\right]\leq q^j\left(s^t\right)\frac{\partial V\left(w,s^t\right)}{\partial w}, \tag{15}$$

with equality whenever $k^j > 0$ and some capital is allocated to the project.

Equation (15) includes two benefits on its left-hand side: an adjusted discounted value generated by the marginal investment and a borrowing capacity change. On the right-hand side, we have the current capital cost of a marginal investment in project j weighted by the marginal value of internal funds at s^t.

The discounted value term includes both the marginal output and the liquidation value of the capital stock and is analogous to discounting the free cash flows from the projects. Importantly, however, it discounts these flows according to the marginal value of resources perceived by the constrained firm and not according to market prices. An additional term incorporates the enhancement in borrowing capacity.

Marginal capital purchases increase a firm's ability to raise external financing to the extent that they increase the available collateral. Because borrowing is limited by commitment problems, these funds are possibly cheaper than internal funds. A premium on borrowing capacity emerges for firms that find themselves against their financing constraints.[20] As a consequence, projects are not only evaluated by the net cash flow increases they induce but also by their ability to attract cheaper collateralized funding. While the APV criterion became notorious in the practice and teaching of applied corporate finance for taking into account the tax advantage of relaxing borrowing capacity, its original formulation allowed for more general benefits, as with the premium on collateralized borrowing that emerges in Equation (15).

In the original APV derivation, decisions to undertake a project are made. In the online appendix, I study an extension that nests that case and also allows some investment decisions to be revised in particular situations.

[20] From equation (9), a binding collateral constraint at s^{t+1}, which limits borrowing at s^t, implies $R^{-1}\frac{\partial V_t\left(s^t\right)}{\partial w_t}-\beta\frac{\partial V_{t+1}\left(s^{t+1}\right)}{\partial w_{t+1}}>0$.

3 Project selection and risk-taking

After the general descriptions of the environment and criteria for evaluating project selection, I study which qualitative consequences are induced by the hedging motive. For this purpose, I analyze two finite-horizon specializations of the model, which illustrate how limited access to external finance changes the firm's risk-taking incentives. In both instances, there are three dates, $t \in \{0, 1, 2\}$. The discount factors are the same for firms and lenders, $\beta = R^{-1} = 1$. Additionally, technologies are described by separable single-factor neoclassical production functions of the following form: $F^j\left(k_t^j, s^t\right) = A^j\left(s^t\right) f\left(k_t^j\right)$, with smooth and strictly concave $f(k) = k^\alpha$ for $\alpha \in (0, 1)$.

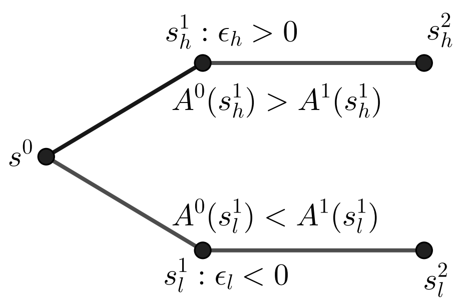
3.1 Persistent productivity

In this section, I specialize the model to illustrate the forces that make constrained firms endogenously distort their risk exposure. The insight that the sign of the hedging demand depends on the correlation of investment opportunities over time shows up in Froot et al. (1993). Rampini and Viswanathan (2013) numerically exemplify that this correlation depends on the persistence of productivity shocks in a dynamic model of risk management.

I analyze how this hedging demand distorts capital budgeting and, more formally, how the sign of the distortion ultimately depends on the interaction between the curvature of the production function and the persistence of TFP shocks. Also, as further explored in the dynamic simulations, standard calibrations suggest that, to facilitate self-financing, constrained firms have incentives to invest in projects with higher exposure to persistent factors that influence productivity and revenues, such as demand and cost shocks. They endogenously choose to have riskier operations than do better-capitalized firms.

Figure 1 summarizes the setup, which is a specialization of the general model. Uncertainty is described by a random variable ϵ, which can take one of two values, $\epsilon \in \{\epsilon_l, \epsilon_h\}$, with $-1 < \epsilon_l < 0 < \epsilon_h$. The shock ϵ has a zero mean and is fully learned at $t = 1$.[21] As a consequence, states at $t = 1$ can be related one-to-one with the realization of ϵ. Each state $s^1 \in \left\{s_l^1, s_h^1\right\}$ has a unique successor, which I label $s^2 \in \left\{s_l^2, s_h^2\right\}$, and $\epsilon\left(s_i^t\right) = \epsilon_i$ for $t = 1, 2$ and $i = l, h$. I study resource allocation across two projects that have different exposures to the underlying state.

Figure 1: Event tree and productivity process



	s^1	s^2
$A^0(s)$:	$\overline{A}\left(1 + \epsilon\left(s^1\right)\right)$	$\overline{A}\left(1 + \epsilon\left(s^2\right)\right)^\rho$
$A^1(s)$:	$\overline{A}\left(1 + \sigma\epsilon\left(s^1\right)\right)$	$\overline{A}\left(1 + \epsilon\left(s^2\right)\right)^\rho$

$\sigma \in (0, 1)$

Project $j = 0$ has a revenue TFP that follows, in logarithms, an autoregressive process of order one

[21] Any residual uncertainty would be inconsequential, because firms pay out all their resources as liquidation dividends at $t = 2$, so I abstract from it.

and persistence ρ, extensively used in the empirical literature. This persistence parameter ρ is key for understanding endogenous risk-taking, because, as I will show, a firm's attitudes toward exposure to the shock ϵ intrinsically depends on it.

Firms have access to another project, $j = 1$, which is safer in the short run, while displaying the same long-run behavior as project $j = 0$. Formally, its productivity at date $t = 1$ is

$$A^1\left(s^1\right) = \overline{A}\left(1 + \sigma\epsilon\left(s^1\right)\right),$$

for $\sigma \in (0, 1)$ and $s^1 \in \left\{s_l^1, s_h^1\right\}$. Notice that the parameter $\sigma < 1$ dampens its short-run risk exposure.

Finally, for simplicity, assume that the capital goods used in both projects are identical, have unit prices, and are fully collateralizable, so that $\theta^0\left(s^t\right) = \theta^1\left(s^t\right) = 1$ always. Also, let depreciation be positive, $\delta > 0$, so that the down payment requirement becomes constant across time and states of the world. Call this common down payment requirement $\wp$.

Notice that by allocating more funds to investment in the safer project, as opposed to the riskier alternative, the firm can reduce its short-run exposure to the shock and, as a consequence, lower the volatility of its cash flows. The first-best capital allocation serves as a useful benchmark, because it equalizes the expected marginal productivities. In this setup, equalizing the expected return simply means making equal capital investments to both projects. Therefore, by comparing quantities invested in each project, one can evaluate the underlying endogenous attitudes toward risk.

The main theoretical result for this section, which will be further explored in the quantitative analysis later, is offered below.

Proposition 1. *Consider the model described above, in which credit-constrained firms allocate a $t = 0$ capital budget across riskier ($j = 0$) and safer ($j = 1$) projects that share a common expected return.*

1. *For sufficiently high net worth, capital allocations are the same across both projects and hence first best, that is, $k_1^0\left(s^0\right) = k_1^1\left(s^0\right)$.*

2. *There exists a threshold $\overline{\rho}\left(\alpha\right) > 0$ for the persistence parameter, such that*

   (a) *whenever $\rho > \overline{\rho}\left(\alpha\right)$, firms invest more in the riskier than in the safer project. That is, $k_1^0\left(s^0\right) \geq k_1^1\left(s^0\right)$, with strict inequality for all constrained firms that invest below the first-best levels at $t = 0$. Consequently, the expected marginal productivity in the safer project is higher than in the riskier project.*

   (b) *whenever $\rho < \overline{\rho}\left(\alpha\right)$, firms invest less in the riskier than in the safer project. That is, $k_1^0\left(s^0\right) \leq k_1^1\left(s^0\right)$, with strict inequality for all constrained firms that invest below the first-best levels at $t = 0$. Consequently, the expected marginal productivity in the safer project is lower than in the riskier project.*

   (c) *the threshold function $\overline{\rho}(\alpha)$ is decreasing in α and satisfies $\overline{\rho}\left(\alpha\right) < 1 - \alpha$. Additionally, as $\sigma \to 1$, $\overline{\rho}\left(\alpha\right) \to 1 - \alpha$.*

The intuition behind Proposition 1 is as follows. Two forces are at play: expected productivity and the scarcity of funds. Together, they shape the marginal value of funds and determine whether resources are more valuable after a positive shock ($\epsilon_h > 0$) or a negative one ($\epsilon_l < 0$).

On one hand, a positive shock raises marginal returns from $t = 1$ onward for any fixed level of capital used. As a consequence, first-best levels of capital investments are higher after positive shocks than after negative ones. This effect is larger the more persistent the shock, because persistence makes returns from additional investment more closely related to the current conditions. This effect is also stronger the closer the production function is to constant returns to scale ($\alpha = 1$), because the parameter α determines the strength of the optimal response of investment to TFP shocks.

On the other hand, cash flows also increase upon a positive productivity shock. As a consequence, firms have more resources to employ after a positive shock than after a negative one. In the presence of decreasing returns to scale, this force makes resources scarcer and more valuable after negative shocks rather than after positive shocks.

Which of these two forces dominates depends on whether the optimal investment response to a productivity shock is higher or lower than the current cash-flow response. Whenever persistence is sufficiently high, firms place a higher value on resources used for funding expansions after a positive shock than on resources used for covering cash-flow shortfalls after negative shocks. Because the riskier project offers a profile of cash flows that is better aligned with the funding needs of a constrained firm, more resources are allocated to it. As a consequence, its expected marginal returns are lower than the ones offered by the safer alternative, which offers a worse matching of flows with the firm's funding needs.

The last item of the proposition also highlights that in this simplified framework, as σ approaches one and the projects become indistinguishable, a simple condition emerges for evaluating whether constrained firms seek increased or decreased risk exposure. Whenever the sum of the elasticity to scale (α) and the persistence parameter (ρ) exceeds one, investment needs are stronger than the cash-flow response and the marginal value of funds covaries positively with the TFP shock.[22] Levels of both the returns to scale and the persistence parameters that are close to one are pervasive in the literature, as will be discussed in Section 4.

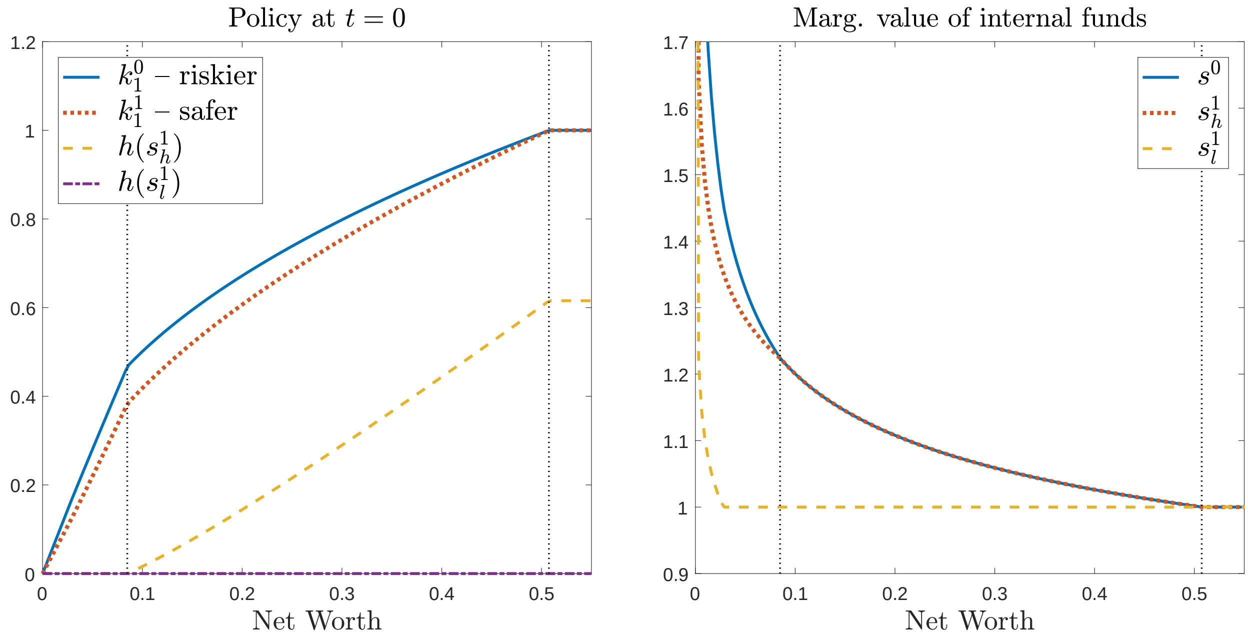
Figure 2 illustrates this situation. On the left-hand side, date $t = 0$ capital budgeting and financial decisions are depicted. Capital investments in the riskier ($j = 0$) and safer ($j = 1$) projects are plotted along with financial slack left for future states. Firms can be split into three categories, as delimited by the vertical dotted lines.

In the leftmost region of the panel, net worth is small. Firms are so constrained that they refrain from using any of their debt capacity for conserving financial slack. Consequently, only distortions in capital budgeting can be used for hedging purposes and these distortions imply larger investments in the riskier project.

In the intermediate region, firms still face binding borrowing constraints. They use both some

[22]If firms could directly control risk exposure at that situation, they would seek increases whenever $\alpha + \rho > 1$, because $\partial_\sigma V\left(w_0, \sigma, s^0\right) \geq 0$. Analogously, firms would value decreased exposure whenever $\alpha + \rho < 1$. For a formal statement and proof, please refer to previous working paper versions of this article.

Figure 2: Persistent shock case



The persistence parameter is $\rho = 0.8$, and the elasticity of returns to scale is $\alpha = 0.9$, so that $\alpha + \rho > 1$. The left-hand side plots capital budgeting and financial policies at date $t = 0$. The right-hand side plots the marginal value of internal funds $(\frac{\partial V_t}{\partial w_t})$ at $t = 0$ and the two possible states at $t = 1$.

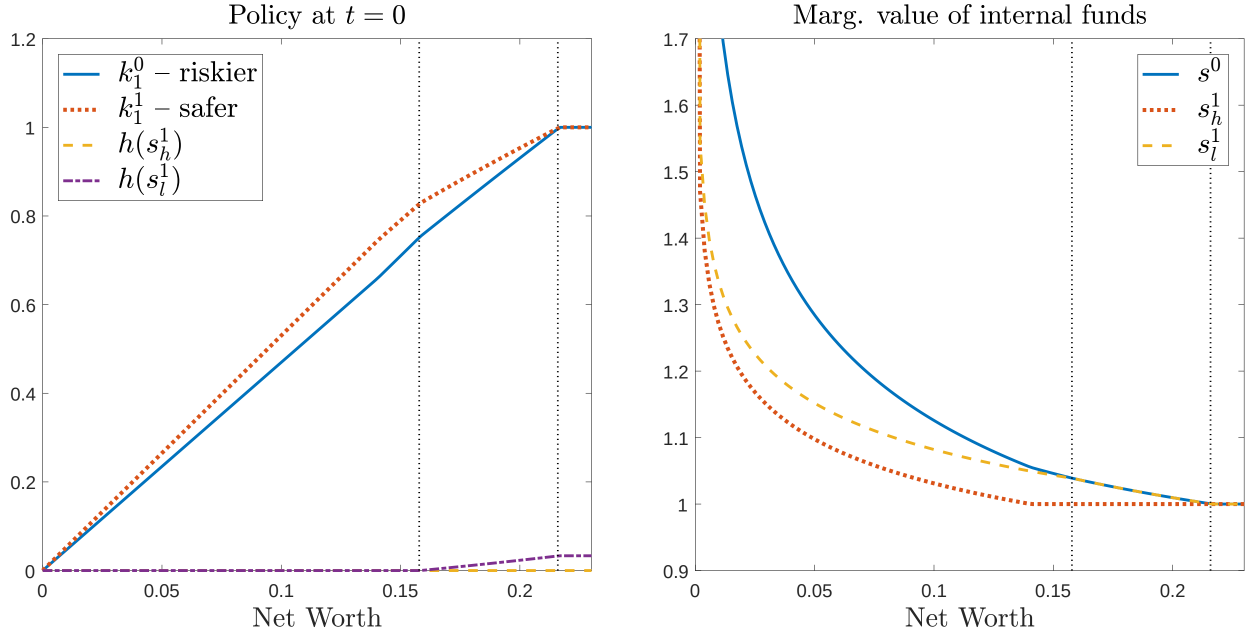
financial instruments and capital budgeting distortions to deal with their hedging demand. Last, in the rightmost region of the panel, firms have enough net worth to fund the first-best scale in all possible situations. Investment is never constrained for these firms and the hedging demand vanishes. As a consequence, capital budgeting is not distorted.

The panel on the right-hand side plots the marginal value of funds at date $t = 0$ and across both $t = 1$ states. Two features are worth noting. First, notice from the ordering of these marginal values that firms are always relatively more constrained at date $t = 0$ than in the growth state $s^1 = \epsilon_h$ at date 1. They are always relatively less constrained at ϵ_l, where they downsize. This rationalizes the direction of the hedging demand, which favors exposure to the underlying shock. Second, as one moves to the right, toward firms that have higher net worth, all marginal returns approach the market rate (normalized to one), whereas their dispersion is reduced. This illustrates a weakening hedging demand for less constrained firms and the ultimate disappearance of the hedging demand for unconstrained ones.

Figure 3 plots the same two panels for a fully transitory shock, with $\rho = 0$. In this case, resources are always more valuable after negative shocks to cash flows, and first-best investment levels do not respond to the shock. The hedging demand points toward reducing exposure and we can see that capital budgeting favors the safer ($t = 1$) alternative.

In Section 4, I revisit the discussion of persistence and endogenous risk-taking for empirically reasonable parameter values in an infinite-horizon quantitative study of the model.

Figure 3: Transitory shock case



The persistence parameter is $\rho = 0$, and the elasticity of returns to scale is $\alpha = 0.9$, so that $\alpha + \rho < 1$. The left-hand side plots capital budgeting and financial policies at date $t = 0$. The right-hand side plots the marginal value of internal funds $(\frac{\partial V_t}{\partial w_t})$ at $t = 0$ and the two possible states at $t = 1$.

3.2 Credit capacity shocks

The environment described in the previous section served to illustrate the connections between the persistence of shocks to productivity and the evaluation of risky cash flows. In the presence of sufficiently persistent shocks, firms do not seek to save resources for the lowest productivity states, because growth concerns outweigh cash-flow insurance. The online appendix documents that smaller firms, typically those understood to be the most financially constrained, show higher volatility not only in their revenues but also in their debt and capital expenditure growth. I also show that this higher volatility is linked to higher exposure to sector-wide fluctuations.

When credit conditions are tight or volatile, capital budgeting and risk-taking decisions can be distorted. Some potential questions follow : Should constrained firms take actions to reduce exposure to fluctuations in credit tightness? Should they conserve internal resources to substitute for external credit that might become scarce? I show that there are some incentives against risk mitigation: firms have reasons to refrain from financial insurance and even, when possible through operational decisions, to increase their exposure to fluctuations in credit conditions.

I explicitly introduce a credit cycle by varying the amount of external funding that can be obtained against the same collateral. The rationale against efforts for risk mitigation is as follows. Shocks to credit capacity (θ) reduce a firm's ability to lever and, consequently, the return it can make on internal funds. Although firms could try to insure against a dry up in credit by conserving its own financial resources for these moments, such situations are naturally low leverage and hence low return. I illustrate how credit- constrained firms can sacrifice net worth and investment levels in low credit capacity states to invest more when credit conditions are more favorable.

I study corporate attitudes toward fluctuations in credit conditions in two stages. First, absent any capital budgeting decisions, I show that constrained firms would fail to use financial instruments to hedge against a possible tightening of borrowing limits. In a second stage, I show that increases in the underlying exposure to credit conditions can actually increase the value of constrained firms. In the airline example, some decisions in capital budgeting, such as whether to use less redeployable aircraft, can arguably increase the underlying exposure to credit fluctuations.

I model these credit fluctuations through a collateralization parameter $\theta\left(s^t\right)$ that depends on the underlying state s^t. For presentation clarity, and to fully isolate the effects from credit constraints and variable leverage, I assume that capital prices and productivity are invariant. Also, I use a single project and drop the dependence of the notation on j. Productivity is constant, $A\left(s^t\right) = A$ and capital prices are normalized to one for all $t,\ s^t \in S^t$. Additionally, the depreciation rate is set to zero, $\delta = 0$, implying that all firms find themselves constrained in all states. Again, this assumption, made for simplicity, avoids the emergence of multiple cases (depending on which states financial constraints bind) and allows me to directly focus on the behavior of non-dividend-paying, financially constrained firms.[23]

Figure 4: Event tree and pledgeability process

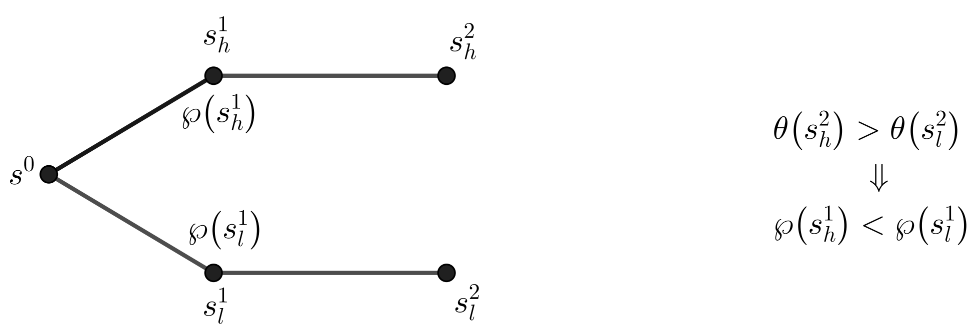


Figure 4 summarizes timing and uncertainty. The realization of one of two possible states is learned at $t = 1$, with $s^1 \in \left\{s_h^1, s_l^1\right\}$. From $t = 1$ toward the final date $t = 2$, the event tree evolves trivially: there is a single successor to each state s^1, to which I refer as $s^2 \in \left\{s_h^2, s_l^2\right\}$. It is worth calling attention to the following feature of timing. The revelation of uncertainty at $t = 1$ determines the recovery rates that apply to $t = 2$ capital, with

$$0 < \theta\left(s_l^2\right) < \theta\left(s_h^2\right) < 1,$$

so that states are ordered by credit conditions and I say that $s_h^1 > s_l^1$. As the recovery rate is already learned at $t = 1$, the state s_h^1 (resp. s_l^1) is a high (resp. low) credit capacity state. A firm that invests k_2 at $t = 1$ can borrow up to $\theta\left(s^2\right) k_2$, implying a down payment requirement that moves in the opposite

[23]With zero depreciation and no discounting, capital investment always dominates dividend payments at the margin at $t = 0, 1$. Financial slack still might be conserved, but dividends are not paid out before $t = 2$. One of the cases that is precluded is that of a sufficient decrease in down payment requirements in s_h^1 that enables some firms, with intermediate levels of net worth, to finance a first-best level of capital in that state, thereby becoming unconstrained. Such would not be the case for sufficiently undercapitalized firms, which are the focus here.

direction relative to credit conditions, because $\wp\left(s_i^1\right)=1-\theta\left(s_i^2\right)$, for $i\in\{h,l\}$.

The variation in the down payment requirement will be shown to be directly responsible for making the marginal return on internal funds higher when credit conditions are more favorable, in line with the intuition given in Section 2.1.[24] For completeness, I also consider a deterministic recovery rate $\theta\left(s^1\right)$, which is known at $t=0$ and applies to $t=1$ capital, no matter which state is realized, and does not play any central role in the analysis that follows. In the same way as recovery rates at date $t=2$ determine down payments at date $t=1$, this recovery rate (which applies at $t=1$) determines the initial ($t=0$) deterministic down payment.

I establish the following proposition.

Proposition 2. *Consider the environment described above.*

1. *Every firm is credit constrained, and the marginal value of funds at* $t=1$ *is given by*

$$\frac{\partial V\left(w\left(s^1\right),s^1\right)}{\partial w\left(s^1\right)}=1+\alpha A\left[w\left(s^1\right)\right]^{\alpha-1}\left[\wp\left(s^1\right)\right]^{-\alpha},$$

   for $s^1\in\left\{s_h^1,s_l^1\right\}$. *As a consequence, under the optimal financial plan, each firm faces marginal values of funds that are increasing in the underlying state, that is,* $\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}>\frac{\partial V\left(w\left(s_l^1\right),s_l^1\right)}{\partial w\left(s_l^1\right)}$, *and never conserves financial slack toward the state with tighter credit conditions, that is,* $h\left(s_l^1\right)=0$.

2. *Capital levels are increasing in the underlying state, that is,* $k_2\left(s_h^1\right)>k_2\left(s_l^1\right)$.

3. *An increase in dispersion of* $\theta\left(s^2\right)$ *across the two states, with preservation of the mean, raises the firm's value ex ante (at* $t=0$*).*

4. *This increase in dispersion additionally leads to increased dispersion of borrowing and capital levels (across states at date* $t=1$*) and of output (at date* $t=2$*).*

The first item of the proposition above shows that the marginal value of resources is higher in high credit capacity states. The rationale for this result is nuanced as two opposing forces are at play. First, with more favorable credit conditions, more financing can be attracted given any amount of collateral. As such, more leverage can be undertaken, operations can be larger, and the larger profits obtained raise the return on internal funds. On the other hand, with more resources invested, decreasing returns to scale imply a lower marginal productivity on investment. This environment illustrates that, for sufficiently financially constrained firms, the first force dominates. Internal funds and credit capacity behave as

[24] The variation in these recovery rates changes how much credit can be obtained against a given quantity of collateral in a way that is unrelated to the collateral's price or output. It is the simplest way to introduce a credit cycle, which is not intrinsically tied to the productivity of an investment. Modeling a credit fluctuation through a change in the recovery rate instead of the prices of capital goods has the advantage of generating an effect which is analytically orthogonal to productivity. The alternative of price fluctuations ties together credit capacity and the expected value of output plus the residual value of the capital stock. Because investment in capital at any date t produces not only output but also capital after depreciation, a change in $q\left(s^{t+1}\right)$ affects the firm's returns and value mechanically.

Although motivating a literal change in a recovery rate along the business cycle is difficult, one can interpret shocks to this variable as any shocks that affect how much a lender is willing to offer against a given amount of collateral. For instance, a deterioration in adverse selection in credit markets would have similar effects.

complements, rather than substitutes. In Section 4.2, I show that the conclusion from this specialization of the model extends to reasonable parametrizations of the firm's dynamic problem.

The flip side of internal funds being more valuable at the margin in high credit capacity states is that their marginal value is lower after credit contractions. Although firms can be inefficiently further from the first-best scale of operation after a credit crunch, a lower ability to borrow and lever up means that the return on one's own resources is depressed. So constrained firms do not have incentives to use financial planning to mitigate the consequences of credit contractions on the scale of their operations. On the contrary, if constrained firms seek any financial assets targeting particular contingencies, the most useful ones are those that pay out when credit conditions are especially favorable, so their payouts can be further levered to fund the expansion of operations.

The third item in the proposition shows that an increase in exposure to credit fluctuations, in the form of a mean-preserving spread of the credit capacity parameter $\theta\left(s^2\right)$, increases the firm's value. So if a constrained firm is given an alternative between two forms of operation that have different underlying exposures to fluctuations in credit conditions, but the same prospects on average, it would choose the most volatile.[25] Naturally, the firm could be willing to pay a premium (or choose slightly less productive forms of operation) for higher exposure. The intuition is as follows. More favorable credit conditions imply higher leverage and, as a consequence, a higher marginal value on resources. A firm more exposed to volatile credit conditions raises relatively more funding exactly when down payment requirements are low and each unit borrowed translates into a larger capital investment.

Additional consequences of this endogenous willingness to seek exposure to fluctuating credit conditions are increased volatility of borrowing, capital expenditures, and output. This channel, therefore, helps rationalize, as an endogenous outcome, the volatile behavior of both real and financial variables of constrained firms.[26]

In sum, potentially important sources of fluctuations in firm growth are volatile credit conditions. I have studied those fluctuations through the introduction of risk in a pledgeability parameter: a reduction in the parameter makes credit tighter by reducing how much can be borrowed against a given amount of collateral. Severely financially constrained firms might fail to have incentives to use financial planning to substitute for thinning credit with their own funds. Given that credit conditions enable leverage, a complementarity in funding emerges: one's own funds are more valuable when using them can attract additional credit. This complementarity amplifies the response of investment, borrowing, and output growth to sources of credit fluctuations.

4 A numerical assessment

In this section, I quantitatively evaluate optimal capital budgeting distortions in the cross section of firms. In the benchmark analysis, constrained firms increase their risk exposure, but distortions are

[25]Possible examples of operational hedging in dealing with exposure to credit fluctuations could be the choice of operating assets (airplanes, e.g.) based on their quality as collateral. Less specific collateral would arguably leave the firm less exposed to credit contractions, but there are no incentives for sacrificing cash-flows or efficiency in production for this exposure reduction.

[26]See the online appendix.

quantitatively mild, except for sufficiently small and constrained firms. In Section 4.1, I conduct a sensitivity analysis, particularly focusing on the importance of the persistence of shocks and technological returns to scale in explaining the direction and magnitudes of capital allocation distortions. Last, I study how fluctuations in credit conditions and the down payment requirement amplify possible risk-taking distortions.

I specialize the model from Section 2 by imposing a recursive structure for uncertainty, a time period of 1 year, and an infinite horizon. I center the numerical analysis on the production function and collateralization parameters obtained in Li et al. (2016) for the airline industry, which are broadly consistent with similar industries. The key productivity process persistence and return to scale parameters are also in line with a literature from a macroeconomic tradition, which typically explores revenue and investment data, while disregarding other financial variables.[27]

Like in Li et al. (2016), I use a three-state Markov chain for describing underlying uncertainty and constructing the firm's TFP process. This chain is obtained from a Tauchen-Hussey approximation to an autoregressive process of order 1, with persistence $\rho = 0.829$ and standard deviation $\sigma = 0.098$. Each production function has a scale elasticity parameter $\alpha = 0.918$. The rate of depreciation for capital goods is set to $\delta = 0.043$, their prices are set to unity, and the degree of collateralization to $\theta = 0.493$. I set the firm's discount factor over delayed dividends to $\beta = 1.05^{-1}$ and the perceived cost of borrowed funds to $R = 1.04$. The wedge between the discount factor and the cost of capital of $\tau = 0.2$ implicitly defined in $R = \beta^{-1}(1 - \tau(1 - \beta))$ is motivated by the tax benefit of debt like in Li et al. (2016).[28]

The following analysis is conducted around the stationary distribution of firms and the intermediate state in the Markov chain. To study distortions in capital budgeting, I endow the firm with two projects that differ solely when business conditions lie in this state, which is the only situation in which both upside and downside risk coexist. By construction, these two projects have the same expected productivity. However, while project $j = 1$ is made riskless in this state, all cash-flow exposure is concentrated in the risky $j = 0$ project.[29] Were a firm to keep its capital budget equalized across these projects, like in the first-best case, it would maximize $t + 1$ expected output and returns. It can, however, increase (resp. mitigate) its short-run risk exposure by investing relatively more in the risky (resp. riskless) project. Under decreasing returns to scale, this is costly in terms of expected output. If a transition occurs, say to the high productivity state, a firm that invested relatively more in the risky project at t

[27] Some of that literature attempts to estimate both TFP processes and returns to scale parameters from firm-level revenue data. Cooper and Haltiwanger (2006) find an elasticity to scale of 0.89 and a persistence of 0.59. Khan and Thomas (2003) identify an elasticity of 0.9 and persistence of 0.92. Midrigan and Xu (2014) document autocorrelations of output of 0.9 over a 1-year horizon and proceed toward a calibration that combines an elasticity to scale of 0.85 and a combination of persistent and transitory shocks. That combination leads to 85% of the cross-sectional variance being accounted for by a fully permanent component, which I take as evidence of the high degree of persistence of TFP in the data. Constant returns to scale are assumed by Collard-Wexler et al. (2011), who study firms across multiple countries and identify a mean persistence of 0.85.

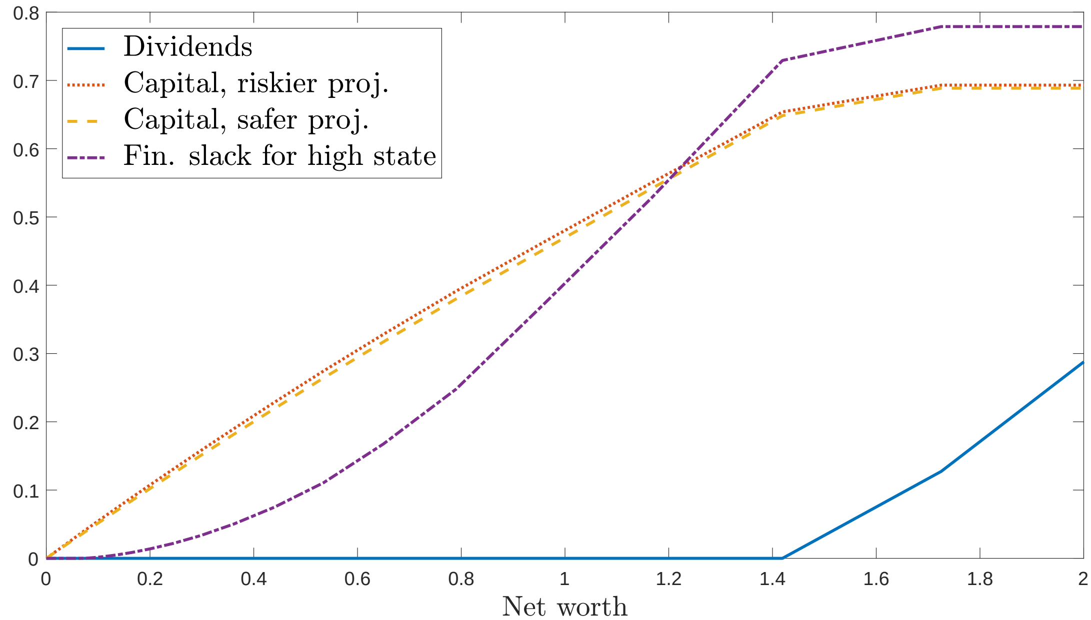
[28] In Li et al. (2016), the discount rate for the firm's own funds is set, prior to the estimation stage, to the riskless rate from U.S. Treasuries. It has been argued that this rate might include relevant liquidity and safety premia and represent too low a benchmark for a riskless rate (see Krishnamurthy and Vissing-Jorgensen (2012), for instance). I set this discount rate to a higher value in the benchmark. For a sensitivity analysis, see Section 4.1.

[29] Formally, let s_l, s_m and s_h denote the three states in increasing order, and let $\log A(s_t)$ be the log-TFP realizations obtained in the discretization above. Whenever $s_t \neq s_m$, $A^0(s_{t+1}) = A^1(s_{t+1}) = A(s_{t+1})$. When $s_t = s_m$, I assume that $A^1_{t+1}(s_{t+1}|s_m) = E[A|s_m]$ and $A^0_{t+1}(s_{t+1}|s_m) = E[A|s_m] + 2(A(s_{t+1}) - E[A|s_m])$. Given that upward and downward transitions have the same probability at $s_t = s_m$, the two projects have the same expected productivity.

collects a cash windfall at $t + 1$.

I first study optimal capital budgeting and financial policies. Firms choose possible dividend payouts, how much capital to allocate to each of the two projects, and how much financial slack to leave to each of the possible states. They have access to a full set of contingent claims, although their borrowing is limited by commitment problems.

Figure 5: Optimal policy as a function of current net worth.

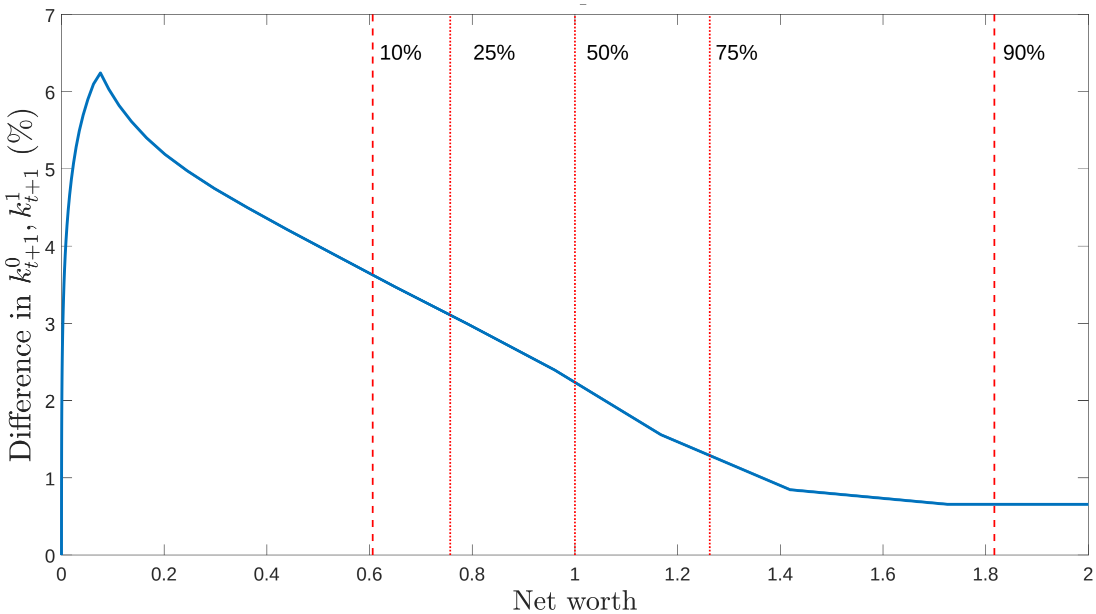


The parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds is $R = 1.04$. The wedge, $\tau = 0.2$, between the discount factor and the cost of capital is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution, while financial variables are normalized by the first-best level of capital.

Figure 5 plots capital budgeting and other financial decisions as a function of net worth, where one unit of net worth is normalized to represent the median value under the stationary distribution. A few properties of the optimal policy are worth emphasizing. First, the direction of hedging indicates that resources are more valuable after positive state innovations than after negative ones. This feature is manifested both through a higher investment in the riskier project and through the conservation of financial slack only to fund expansion in the highest state. Financial risk management is not used at all for a range of firms with extremely low values of net worth and changes quickly in magnitude along the cross section of possible net worth levels. Similarly, only sufficiently large firms pay out dividends.

From Figure 5, the distortion in favor of the riskier project (while still noticeable) is dwarfed by the scale of the financial decisions. Figure 6 further explores the magnitude of this distortion in the cross section of firms. The vertical lines represent key percentiles of the stationary firm distribution. The median firm invests approximately 2.2% more in the riskier technology, bringing its marginal return below the one for the riskless project. For a firm at the 5th percentile of the stationary distribution, this distortion is 3.86%. By distorting investment from expected marginal product equalization, firms

Figure 6: Distortion in capital allocation



The parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q^j_t = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds is $R = 1.04$. The wedge, $\tau = 0.2$, between the discount factor and the cost of capital is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution.

increase their exposure to the risk factor that shapes revenues, thereby creating endogenous volatility. Figure 5 also can be understood as describing the shape of the incentives to take on endogenous volatility, which would peak at a very low level of net worth (about 10% of the median firm) and gradually recede as firms grow larger.

Larger values for this distortion, above 6%, are reached by firms with a net worth that is only a small fraction of the median level. This situation, although unlikely under the stationary distribution, might apply to firms in an entry phase. Also, for these low net worth values, a non-monotonicity in the hedging-induced distortion is displayed. The economic intuition is as follows. For these firms, returns are very high, and fast growth occurs even in the lowest of productivity states. In the same way that smaller firms refrain from using financial instruments for hedging, as using them would lower investment levels and impede growth, the low net worth firms are less willing to sacrifice any growth to obtain hedging benefits. Interestingly, this non-monotonicity is only a feature of the fully dynamic environment, in which firms can postpone hedging for future dates, but is not present in the short-horizon setup of Section 3.1.

The capital allocation distortion is monotonic around net worth values that are frequent under the stationary distribution: it decreases for larger firms, but does not disappear, reaching a plateau at around 0.66%. Because borrowed funds come with a premium, firms never overcome the threat of eventually binding borrowing constraints, and the hedging motive does not entirely vanish, despite the fact that a maximum scale is reached and dividends are paid out after a sufficiently long sequence of good shocks.

Although we can identify a meaningful discrepancy in capital allocation, it is significantly smaller than what would be induced by heterogeneity in expected profitability. For instance, in the first-best case, a difference of 1% in expected TFP across the two projects would lead to a 12% difference in capital allocation. Therefore, a takeaway from this numerical experiment is that, while hedging concerns are present, they are still expected to be secondary in comparison to expect return differences. One reason is that firms have several instruments available for dealing with their hedging demand, such as dividend postponement, distortions in capital budgeting, and financial risk management. Section OA.3, in the online appendix, evaluates how an additional friction, in the form of incompleteness in the set of financial instruments available for hedging, by removing one of these instruments, amplifies distortions in capital budgeting. Also, in the next section, I evaluate how different features of the environment affect the hedging motive and the relative costs of the different instruments that target it.

4.1 Returns to scale, persistence, and other features of the environment

In this section, I describe how the magnitude of the capital allocation distortion responds to key parameters in the environment. The key conclusions are that distortions favoring riskier projects are increasing in the persistence of productivity shocks, the returns-to-scale parameter, and the (tax-induced) wedge between discount rates and borrowing costs.

As anticipated in Section 3.1, returns to scale play a key role in shaping how strongly optimal investment needs respond to shocks in business conditions. The closer to constant returns the technology is, the larger the incentives to fund expansions after positive innovations. Returns to scale also determine how costly it is to use capital budgeting distortions for risk management: the more strongly decreasing these returns to scale are, the more costly in terms of expected output it becomes to depart from first-best capital shares.

Figure 7a illustrates noticeable distortions in capital allocation whenever the scale elasticity parameter, α, exceeds 0.75. Around this value, effects are mostly salient for very low quantiles of the stationary distribution of firms, as better-capitalized firms use more financial risk management. They increase in magnitude along all quantiles and eventually diverge to infinity, as the technology gets closer to constant returns to scale ($\alpha = 1$). Sensitivity to parameters is moderate around $\alpha = 0.9$, and any effects essentially disappear for $\alpha < 0.7$.

Another key parameter in the previous analysis described the persistence in business conditions. The more persistent shocks are, the stronger the hedging motive is. From the initial finite horizon theory, we also expected a sign change in the direction of this hedging distortion. Highly persistent shocks make sure that current conditions predict future profitability and investment needs. In this case, exposure to a factor that shapes current conditions helps fund future investment. Purely transitory shocks, on the other hand, only generate fluctuations in net worth, which are costly in the presence of imperfect financing, and should be mitigated.

Figure 7b plots optimal distortions along the stationary distribution for different values of persistence in the productivity process. Although the sign reversal, which is expected in theory, occurs around a persistence level of $\rho = 0.05$, it is barely visually or economically perceptible. The main lesson from this sensitivity analysis is as follows: for the benchmark case, firms have adequate alternative instruments for dealing with transitory shocks, such as preemptive net worth accumulation and contingent financial contracts. Here, for high levels of persistence two other features grab attention. First, as current conditions become better predictors of future investment needs, effects increase. Second, the distribution of distortions fans out, indicating that the cross-sectional heterogeneity in the magnitude of distortions becomes more pronounced as persistence increases.

Figures 8c and 8d plot the consequences of changes in the borrower's discount rate and the wedge between that rate and the perceived cost of collateralized funding. Regarding the wedge, which is parametrized like in Li et al. (2016), we observe a large sensitivity to small values of τ but decreasing effects that roughly stabilize for values above $\tau = 0.2$. The discount factor for the delayed dividends shows some sensitivity to values of $\beta > 0.95$. For instance, distortions decrease to roughly half their values if one moves from the benchmark case to $\beta = 0.98$. A large fraction of this sensitivity originates from the tax benefit interpretation of the wedge in the cost of funds and the implied parametrization. In the model, borrowing against collateral seems relatively cheap to firms and, as a mirror image, saving (or leaving financial slack for risk management purposes) relatively expensive. How so depends on the level difference between the lender's discount factor (R) and the borrower's (β). Under the Li et al. (2016) parametrization, this distance decreases fast as β approaches one. As a consequence, risk management through financial instruments becomes relatively cheap in that limit, firms accumulate assets in the stationary distribution, and any capital budgeting distortions are greatly reduced.[30]

I also study the consequences of different levels of underlying volatility and the recovery value of collateral. Figure 8e plots the sensitivity analysis regarding the baseline dispersion in productivity. The most noticeable feature is a lack of monotonicity, with a peak level of distortions being identified for each quantile. Also, lower quantiles, representing more constrained firms, are more sensitive to changes in underlying volatility. From additional inspection of the underlying policies and stationary distribution, I conclude that once firms are subject to higher underlying volatility, they both increase the distortions in capital budgeting and delay dividends, opting to conserve more financial slack. This occurs at any fixed level of net worth. As a consequence, higher volatility shifts the stationary distribution of firms toward the right. This increase in the firm's scale along all key quantiles dominates the direct effect of more capital budgeting distortions for sufficiently high levels of volatility, implying that lower distortions are perceived in the stationary cross section.

Regarding the recovery value of collateral, the level of distortions in the cross section changes little. As the recovery rate increases, the stationary net worth distribution is mostly stable, and the capital levels approach the first-best level. The stationary distribution only starts to shift to the left for extremely high levels, such as $\theta > 0.9$, but, under that situation, capital is almost perfectly pledgeable and firms rarely suffer a string of bad results that is long enough to make them constrained again. As a consequence, we observe a drop in distortions in capital budgeting that is induced by hedging demand.

[30] If one studies an alternative parametrization with a constant additive wedge, which can be motivated by a setting in which stores of value used for backing liquidity carry a premium (like in Holmström and Tirole, 1998, 2011), most of this sensitivity disappears.

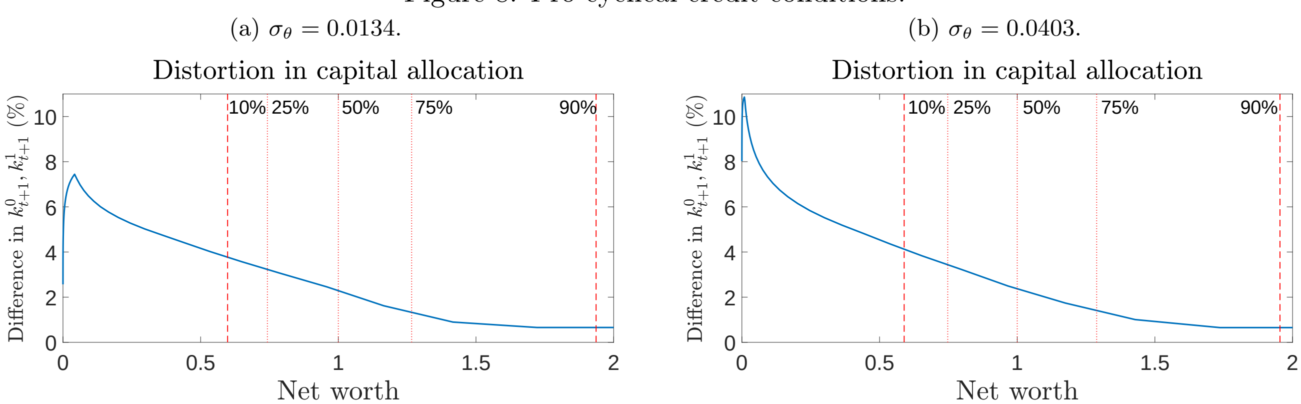
4.2 Fluctuations in credit conditions, capital prices, and the down payment requirement

The direction and intensity of the hedging motive are driven by the endogenous behavior of the marginal value of internal funds. Section 2.1 has shown that credit conditions play a key role in this, as a relaxation of credit constraints allows more leverage to be undertaken and helps increase the return on own funds. To the best of my knowledge, this effect is new to the literature and has not been previously evaluated in a dynamic model. In this section, I show that pro-cyclical credit conditions can amplify incentives for endogenous risk-taking by constrained firms.

The simplest way to introduce pro-cyclical credit conditions in this framework is by allowing $\theta^j(s_t)$ to vary with the underlying state, which, in the dynamic simulations to now, has only affected cash flows. I make this collateralization variable pro-cyclical, so that feasible leverage increases with productivity and revenues. To make matters simple, I keep the exposure to credit conditions constant across both projects, so distortions in investment from symmetry are purely driven by the projects' heterogeneous exposure to cash-flow risk.[31]

Figure 8 plots the distortions in capital budgeting across both projects for two levels of volatility in credit conditions and can be directly compared to Figure 6, where no volatility appears. The main conclusion is that the pro-cyclical credit conditions can lead to a more pronounced hedging demand.[32] Additional consequences from an increase in volatility of credit conditions include a small reduction of the median net worth (about 2%) and an increase in the firm-size dispersion. Both effects are consistent with firms endogenously increasing their risk exposure.

Figure 8: Pro-cyclical credit conditions.



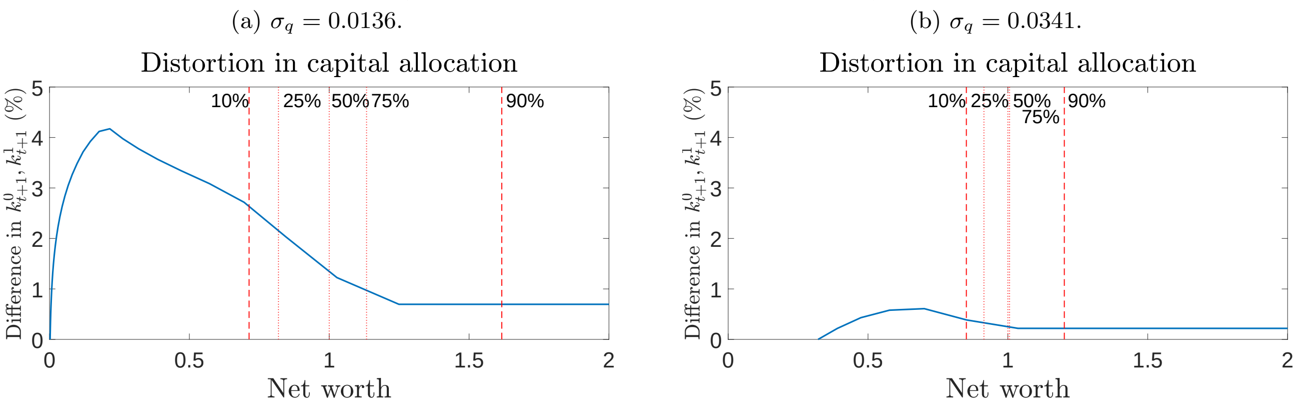
Capital price fluctuations have an opposite consequence. Figure 9 plots the distortions in capital budgeting caused when the price of the capital goods is assumed to be pro-cyclical (the pledgeability

[31]Formally, I allow $\theta(s_t)$, where $s_t \in \{s_l, s_m, s_h\}$ are the three states. While $\theta(s_m) = 0.493$, like in the baseline calibration, a mean-preserving spread that sets $\theta(s_h) = \theta(s_m) + \Delta$ and $\theta(s_l) = \theta(s_m) - \Delta$ is introduced. The parameter σ_θ describes the unconditional standard deviation of this stochastic θ.

[32]Notice there exist forces in opposing directions after a (positive) shock. On the one hand, higher cash flows and increased leverage help firms scale up and mitigate the effects of financial constraints. On the other hand, higher leverage itself and higher productivity combined increase returns on internal funds. In the neighborhood of parameter values under study, the forces in the latter direction dominate.

parameter is reverted to acyclical, like in the benchmark).[33] The consequences are a mirror image of the pro-cyclical relaxation of credit constraints. The hedging motive is dampened and essentially disappears in panel 9b, whereas firm size distributions are compressed because of the mean reversion in the economic conditions embedded in the Markov chain. As prices can only go down after a positive shock, the future value of the capital stock becomes significantly lower than the current price, increasing down payment requirements and making it more difficult to finance expansions. As a consequence, returns on investment become less exposed to current conditions. In panel 9b, we see evidence of an almost acyclical return on internal funds.

Figure 9: Pro-cyclical prices of capital goods



For sufficiently volatile capital good prices above the levels plotted in the second panel, a sign reversal of that return occurs. Once that happens, because of strongly pro-cyclical down payment requirements, a change in the direction of the hedging demand follows. This extreme scenario featuring countercyclical returns seems unlikely both in a theoretical general equilibrium framework and empirically.

The main lesson from these exercises is that the cyclical behavior of down payment requirements, which are affected by both prices and credit conditions, can influence the direction and magnitude of the hedging motive. In particular, strongly countercyclical down payment requirements can greatly increase incentives for risk-taking. Further empirical study of the cyclicality of these variables can offer improved guidance on the expected magnitude of the distortions.

5 Conclusion

Without perfect access to credit, firms cannot fully rely on outside funding to invest efficiently. This gives rise to a hedging motive, which is reflected on both the financial and investment sides of corporate decisions.

I have paid special attention to the induced investment distortions. For this, I have extended a standard model of corporate investment and financing with endogenous collateral constraints to incorporate

[33] In a way analogous to the previous exercise, the state $s_t \in \{s_l, s_m, s_h\}$ determines both productivity and capital prices (and hence down payment requirements). Prices are such that $q(s_m) = 1$ as before, whereas a mean-preserving spread is introduced by setting $q(s_h) = 1 + \Delta$ and $q(s_l) = 1 - \Delta$. The parameter σ_q describes the unconditional standard deviation of the stochastic capital price.

explicit capital budgeting. The model offers a unified framework for studying the factors that shape the value of a corporation's internal funds and the interaction of alternative hedging instruments. It abstracts from other potentially relevant elements, such as internal agency problems, adjustment costs, and indivisibilities. The analytical insights derived, however, are expected to extend to related frameworks and, potentially, interact with additional frictions.

While the hedging motive creates a desire to smooth out transitory cash-flow fluctuations, empirically reasonable levels of persistence in productivity shocks cause constrained firms to be more willing to bear risks that correlate with their productivity processes. Therefore, because this form of risk-taking facilitates self-financing, I have illustrated a channel through which more constrained firms become endogenously more volatile.

I have also emphasized the importance of leverage for optimal hedging. Leverage makes internal funds complementary to external credit and can make resources more valuable to the firm when credit conditions are more favorable. As a consequence, constrained firms might show a risk-taking attitude toward their exposure to aggregate credit conditions.

Distortions in capital budgeting and the use of standard financial instruments for hedging are imperfect substitutes. I have shown that the costlier financial hedging becomes as an alternative, the more distortions one observes in capital budgeting. For instance, while financial markets might fail to offer products contingent on the factors that shape a particular firm's investment opportunities, real investment projects, by their own nature, are likely to offer exposure to these contingencies.

Finally, future empirical work can help test and quantify the importance of capital budgeting distortions and further describe their interactions with cross-sectional heterogeneity and the evolution of financial market instruments.

A Proofs omitted from the main text

Proof of Proposition 1.1. In the first-best case, with $\beta = R = 1$, the timing of dividends is irrelevant for optimality. Optimal capital investments are separable across time and projects, so one can solve the simple $\max_{k^j_{t+1}} E\left[f\left(k^j_{t+1}\right) + (1-\delta)\, k^j_{t+1}|s^t\right] - k^j_{t+1}$ for each $j \in \{0,1\}$ and s^t, for $t = 0, 1$. The first-order conditions at s^0 imply $\alpha E_0\left[A^j\left(s^1\right)\right] k^j_1\left(s^0\right)^{\alpha-1} = \delta$. As $E_0\left[A^0\left(s^1\right)\right] = E_0\left[A^1\left(s^1\right)\right]$, it follows that $k^0_1\left(s^0\right) = k^1_1\left(s^0\right)$.

□

Proof of Proposition 1.2. First, as an intermediate step, I characterize the value function at $t = 1$, which can be simply written as

$$V\left(w, s^1\right) = \max_{\{k^j\}_j, h \geq 0} \sum_j A^j\left(s^2|s^1\right) f^j\left(k^j\right) + h\left(s^2|s^1\right),$$

s.t. $\sum_j \wp k^j + h\left(s^2|s^1\right) \leq w$. Here, value is not compromised in setting $d\left(s^1\right) = 0$, given that s^2 is terminal, leading to a liquidation dividend, and $R = \beta = 1$. Given $A^0\left(s^2\right) = A^1\left(s^2\right) = \overline{A}\left(1 + \epsilon\left(s^2\right)\right)^\rho$, optimal capital deployments are the same across both projects, and it follows that

$$V\left(w,s^{1}\right)=\begin{cases}2^{1-\alpha}A^{j}\left(s^{2}\right)\left(\frac{w}{\wp}\right)^{\alpha}, & if\, w\leq w^{*}\left(s^{1}\right)\\ V\left(w^{*}\left(s^{1}\right),s^{1}\right)+w-w^{*}\left(s^{1}\right), & if\, w>w^{*}\left(s^{1}\right),\end{cases}$$

where $w^{*}\left(s^{1}\right)$ solves $\alpha 2^{1-\alpha}A^{j}\left(s^{2}|s^{1}\right)\left(\frac{w}{\wp}\right)^{\alpha-1}=1$, making the firm effectively unconstrained.
Again, since $\beta=R^{-1}$, value is not lost by requiring $d\left(s^{0}\right)=0$, which eliminates some cases of indeterminacy of the optimal policy. Then the value function at s^{0} simply can be written as

$$V\left(w_{0},s^{0}\right)=\max_{\{k^{j}\}_{j},\{h(s^{1})\}_{s^{1}}\geq 0}E\left[V\left(w\left(s^{1}\right),s^{1}\right)\right] \tag{16}$$

s.t. $w\left(s^{1}\right)=\sum_{j}A^{j}\left(s^{1}\right)\left(k^{j}\right)^{\alpha}+h\left(s^{1}\right)$ and $w_{0}=\wp\sum k^{j}+E\left[h\left(s^{1}\right)\right].$

□

Concavity can be easily verified. Taking the first-order conditions and using the envelope theorem, one gets

$$k^{j}:\ E\left[A^{j}\left(s^{1}\right)\frac{\partial V\left(w\left(s^{1}\right),s^{1}\right)}{\partial w}\right]\alpha\left(k^{j}\right)^{\alpha-1}-\wp\frac{\partial V\left(w_{0},s^{0}\right)}{\partial w}=0 \tag{17}$$

and

$$h\left(s^{1}\right):\frac{\partial V\left(w\left(s^{1}\right),s^{1}\right)}{\partial w}-\frac{\partial V\left(w_{0},s^{0}\right)}{\partial w}\leq 0,\text{ with equality if }h\left(s^{1}\right)>0. \tag{18}$$

I first define an auxiliary function

$$\varphi\left(k^{0},k^{1},s^{1}\right):=\alpha 2^{1-\alpha}\left(\frac{\overline{A}}{\wp}\right)^{\alpha}\left(1+\epsilon\left(s^{1}\right)\right)^{\rho}\left(\left(1+\epsilon\left(s^{1}\right)\right)k^{0}+\left(1+\sigma\epsilon\left(s^{1}\right)\right)k^{1}\right)^{\alpha-1},$$

which represents $\partial_{w}V\left(w\left(s^{1}\right),s^{1}\right)$ as a function of $k^{0}\left(s^{0}\right)$ and $k^{1}\left(s^{0}\right)$ whenever $h\left(s^{1}\right)=0$ and $\partial_{w}V\left(w\left(s^{1}\right),s^{1}\right)>1$. I also define $\overline{k}:=\frac{k^{0}+k^{1}}{2}$ as the average capital allocation across projects. I then analyze three possible, but mutually exclusive, characterizations of a solution to Program (16).

1. $\partial_{w}V\left(w\left(s_{h}^{1}\right),s_{h}^{1}\right)>\partial_{w}V\left(w\left(s_{l}^{1}\right),s_{l}^{1}\right)\geq 1$.
   It follows from the first-order conditions for financial slack that $h\left(s_{l}^{1}\right)=0$, and with respect to capital that $k^{0}>k^{1}$. Then, since $h\left(s^{1}\right)\geq 0$, $\partial_{w}V\left(w\left(s_{h}^{1}\right),s_{h}^{1}\right)\leq\varphi\left(k^{0},k^{1},s^{1}\right)\leq\varphi\left(\overline{k},\overline{k},s_{h}^{1}\right)$. Analogously, $\partial_{w}V\left(w\left(s_{l}^{1}\right),s_{l}^{1}\right)\geq\varphi\left(k^{0},k^{1},s_{l}^{1}\right)\geq\varphi\left(\overline{k},\overline{k},s_{l}^{1}\right)$. Combining both, one gets $\varphi\left(\overline{k},\overline{k},s_{h}^{1}\right)>\varphi\left(\overline{k},\overline{k},s_{l}^{1}\right)$ which simplifies to
   $$\left(1+\epsilon_{h}\right)^{\rho}\left(\left(1+\epsilon_{h}\right)+\left(1+\sigma\epsilon_{h}\right)\right)^{\alpha-1}>\left(1+\epsilon_{l}\right)^{\rho}\left(\left(1+\epsilon_{l}\right)+\left(1+\sigma\epsilon_{l}\right)\right)^{\alpha-1}. \tag{19}$$
   Additionally, $\partial_{w}V\left(w_{0},s^{0}\right)>1$ follows from 18 and, together with 17, implies that k^{0},k^{1} are strictly below the first-best level.

2. $\partial_{w}V\left(w\left(s_{l}^{1}\right),s_{l}^{1}\right)>\partial_{w}V\left(w\left(s_{h}^{1}\right),s_{h}^{1}\right)\geq 1$. All steps are analogous to the previous computations, and one gets
   $$\left(1+\epsilon_{h}\right)^{\rho}\left(\left(1+\epsilon_{h}\right)+\left(1+\sigma\epsilon_{h}\right)\right)^{\alpha-1}<\left(1+\epsilon_{l}\right)^{\rho}\left(\left(1+\epsilon_{l}\right)+\left(1+\sigma\epsilon_{l}\right)\right)^{\alpha-1} \tag{20}$$
   instead of (19).

3. $\partial_w V\left(w\left(s_h^1\right), s_h^1\right) = \partial_w V\left(w\left(s_l^1\right), s_l^1\right)$. This case has two mutually exclusive sub-cases. In both of these sub-cases, we have $k^0 = k^1 = \overline{k}$ as a consequence of the capital investment Euler equation.

   (a) $\partial_w V\left(w_0, s^0\right) = \partial_w V\left(w\left(s_h^1\right), s_h^1\right) = \partial_w V\left(w\left(s_l^1\right), s_l^1\right)$. From 17, it follows that for $j = 0, 1$, $\alpha \overline{A}\left(k^j\right)^{\alpha-1} = \delta$, that is, capital reaches the first-best level at $t = 0$.

   (b) $\partial_w V\left(w_0, s^0\right) > \partial_w V\left(w\left(s_h^1\right), s_h^1\right) = \partial_w V\left(w\left(s_l^1\right), s_l^1\right)$. It follows that $h\left(s_h^1\right) = h\left(s_l^1\right) = 0$. Then $V\left(w\left(s^1\right), s^1\right) = \max\left\{1, \varphi\left(\overline{k}, \overline{k}, s^1\right)\right\}$ for $s^1 = s_h^1, s_l^1$, which requires

$$(1+\epsilon_h)^{\rho}\left((1+\epsilon_h) + (1+\sigma\epsilon_h)\right)^{\alpha-1} = (1+\epsilon_l)^{\rho}\left((1+\epsilon_l) + (1+\sigma\epsilon_l)\right)^{\alpha-1}. \tag{21}$$

   Additionally, $\partial_w V\left(w_0, s^0\right) > \partial_w V\left(w\left(s^1\right), s^1\right)$ implies k^0,k^1 are below the first-best level, that is, $\alpha \overline{A}\left(k^j\right)^{\alpha-1} > \delta$.

Together, the cases above exhaust all possibilities for the behavior of a solution and provide a key condition for evaluating whether the marginal value of resources tends to be higher in s_h^1 or s_l^1. I can define a useful function in

$$\psi_\sigma(\rho, \alpha) := \left(\frac{1+\epsilon_h}{1+\epsilon_l}\right)^{\rho}\left[\frac{(1+\epsilon_l) + (1+\sigma\epsilon_l)}{(1+\epsilon_h) + (1+\sigma\epsilon_h)}\right]^{1-\alpha} - 1,$$

which is a continuous function in $[0,1]^2$ and is increasing in both ρ and α. It also satisfies $\psi(\rho = 0, \alpha) < 0$ and $\psi(\rho = 1, \alpha) > 0$ for all $\alpha \in [0,1]$. Therefore, I can implicitly define $\psi(\overline{\rho}(\alpha), \alpha) = 0$, so that whenever $\rho > \overline{\rho}(\alpha)$ (respectively, $<$ or $=$), condition 19 is ensured (respectively, 20 or 21). Also, $\psi_\sigma(1-\alpha, \alpha) > 0$ implies $\overline{\rho}(\alpha) < 1 - \alpha$. Last, as $\sigma \to 1$, $\psi_\sigma \to \left(\frac{1+\epsilon_h}{1+\epsilon_l}\right)^{\rho+\alpha-1} - 1$ so $\overline{\rho}(\alpha) \to 1 - \alpha$.

Proofs from Section 3.2

Proof of Proposition 2.

First, I present a characterization of the value function, which is instrumental to the proofs of the items that follow. Given that $t = 2$ is a terminal date, liquidation dividends are paid out and $\frac{\partial V\left(w\left(s^2\right), s^2\right)}{\partial w\left(s^2\right)} = 1$. We have then that

$$\frac{\partial V\left(w\left(s^1\right), s^1\right)}{\partial w\left(s^1\right)} = \min\left\{\frac{\alpha A\left(\frac{w\left(s^1\right)}{\wp\left(s^1\right)}\right)^{\alpha-1} + \left(1 - \theta\left(s^2|s^1\right)\right)}{\wp\left(s^1\right)}, 1\right\}$$

for $s^1 = s_h^1, s_l^1$, and $s^2|s^1$ being its unique successor. The second case in braces would occur whenever dividends are paid out. Given that $\wp\left(s^1\right) = 1 - \theta_2\left(s^2|s^1\right)$, it simplifies to $\frac{\partial V\left(w\left(s^1\right), s^1\right)}{\partial w\left(s^1\right)} = 1 + \alpha A\left[w\left(s^1\right)\right]^{\alpha-1}\left[\wp\left(s^1\right)\right]^{-\alpha} > 1$. This expression is decreasing in both $\wp\left(s^1\right)$ and $w\left(s^1\right)$. As a consequence, it is increasing in $\theta\left(s^2\right)$ and the weak inequality $\frac{\partial V\left(w\left(s_h^1\right), s_h^1\right)}{\partial w\left(s_h^1\right)} \geq \frac{\partial V\left(w\left(s_l^1\right), s_l^1\right)}{\partial w\left(s_l^1\right)}$ follows.

1. Proof that $\frac{\partial V\left(w\left(s_h^1\right), s_h^1\right)}{\partial w\left(s_h^1\right)} > \frac{\partial V\left(w\left(s_l^1\right), s_l^1\right)}{\partial w\left(s_l^1\right)} > 1$ and $h\left(s_l^1\right) = 0$.

Consider two cases: $h\left(s_h^1\right)=0$ and $h\left(s_h^1\right)>0$. In the former, $w\left(s_h^1\right)=Ak_0^\alpha+\left(1-\theta_1\left(s^0\right)\right)k_0\leq w\left(s_l^1\right)$. Then $\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}=1+\alpha A\left[w\left(s_h^1\right)\right]^{\alpha-1}\left[\wp\left(s_h^1\right)\right]^{-\alpha}\geq 1+\alpha A\left[w\left(s_l^1\right)\right]^{\alpha-1}\left[\wp\left(s_h^1\right)\right]^{-\alpha}>1+\alpha A\left[w\left(s_l^1\right)\right]^{\alpha-1}\left[\wp\left(s_l^1\right)\right]^{-\alpha}=\frac{\partial V\left(w\left(s_l^1\right),s_l^1\right)}{\partial w\left(s_l^1\right)}$. In the latter, one needs $\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}=\frac{\partial V\left(w_0,s^0\right)}{\partial w_0}\geq\frac{\partial V\left(w\left(s_l^1\right),s_l^1\right)}{\partial w\left(s_l^1\right)}$. If the equality between all three marginal values were to happen, the capital investment Euler equation would establish that $\frac{\partial V\left(w_0,s^0\right)}{\partial w_0}=\left(1+\alpha A\left[w_0\right]^{\alpha-1}\left[\wp_0\right]^{-\alpha}\right)E\left[\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}\right]\implies 1=1+\alpha A\left[w_0\right]^{\alpha-1}\left[\wp_0\right]^{-\alpha}>1$ reaching a contradiction.
Therefore, I show that $\frac{\partial V\left(w_0,s^0\right)}{\partial w_0}\geq\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}>\frac{\partial V\left(w\left(s_l^1\right),s_l^1\right)}{\partial w\left(s_l^1\right)}>1$. From the Euler equation for $h\left(s_l^1\right)$, it follows that $h\left(s_l^1\right)=0$.

2. $k_2\left(s_h^1\right)>k_2\left(s_l^1\right)$.

   Because $\frac{\partial V\left(w\left(s^1\right),s^1\right)}{\partial w\left(s^1\right)}>1$ for $s^1=s_h^1,s_l^1$, we get the result that firms resort to maximal leverage at $t=1$ and $k_2\left(s_h^1\right)\geq\frac{Ak_1^\alpha(s_0)+(1-\theta_1)k_1^\alpha(s_0)}{\wp\left(s_h^1\right)}>\frac{Ak_1^\alpha(s_0)+(1-\theta_1)k_1(s_0)}{\wp\left(s_l^1\right)}=k_2\left(s_l^1\right)$ where the last equality follows from the fact that $\frac{\partial V\left(w_0,s^0\right)}{\partial w_0}\geq\frac{\partial V\left(w\left(s_h^1\right),s_h^1\right)}{\partial w\left(s_h^1\right)}>\frac{\partial V\left(w\left(s_l^1\right),s_l^1\right)}{\partial w\left(s_l^1\right)}$, which ensures that $h\left(s_l^1\right)=0$.

3. Proof that a mean-preserving spread of $\theta\left(s^2\right)$ raises the firm's value ex ante (at $t=0$).

   I will resort to a parametric approach for studying increases in the dispersion of pledgeability, fixing the two-state structure and their probabilities. Any mean-preserving spread of $\theta\left(s^2\right)$ within this fixed two-state structure can be described as a parameter of $\gamma\in\Gamma:=\left(0,\pi\left(s_h^1\right)\left(1-\theta\left(s_h^2\right)\right)\right)$, such that

$$\theta\left(\gamma,s_h^2\right)=\theta\left(s_h^2\right)+\frac{\gamma}{\pi\left(s_h^1\right)}\text{ and }\theta\left(\gamma,s_l^2\right)=\theta\left(s_h^2\right)-\frac{\gamma}{\pi\left(s_l^1\right)}.$$

   By construction, changes in γ keep the mean unchanged. I now move to the firm's problem for any possible value of the mean-preserving spread parameter of γ. Given the previous items, which did not depend on the dispersion of the pledgeability parameter, it follows that for each possible γ

$$V\left(w,\gamma,s^1\right)=Aw^\alpha\left(1-\theta\left(\gamma,s^2|s^1\right)\right)^{-\alpha}+w,$$

   for $s^1\in\left\{s_h^1,s_l^1\right\}$. This function is increasing and convex in $\theta\left(\gamma,s^2\right)$, which can be verified by first and then double differentiation. The time $t=0$ value function is then the solution to

$$V\left(w_0,\gamma,s^0\right)=\max_{k_0\geq 0,\{h(s^1)\geq 0\}_{s^1}}E\left[V\left(w\left(s^1\right),\gamma,s^1\right)\right]$$

   s.t.

$$w\left(s^1\right)=Af\left(k_0\right)+\left(1-\theta\left(s^1\right)\right)k_0+h\left(s^1\right)$$

$$w_0=\wp_0k_0+\sum\pi\left(s^1\right)h\left(s^1\right).$$

   From the previous item, it follows that the solution features $h\left(s_h^1\right)\geq h\left(s_l^1\right)=0$. Notice first that, using the envelope theorem, for every $\gamma \in \Gamma$

$$\frac{\partial V\left(w_0,\gamma,s^0\right)}{\partial \gamma} = E\left[Aw\left(s^1\right)^{\alpha}\left(1-\theta\left(\gamma,s^2|s^1\right)\right)^{-\alpha-1}\frac{\partial\theta\left(\gamma,s^2\right)}{\gamma}\right]$$
$$= AE\left[\frac{k\left(\gamma,s^1\right)^{\alpha}}{1-\theta\left(\gamma,s^2|s^1\right)}\frac{\partial\theta\left(\gamma,s^2\right)}{\gamma}\right] = A\left[\frac{k\left(\gamma,s_h^1\right)^{\alpha}}{1-\theta\left(\gamma,s_h^2\right)} - \frac{k\left(\gamma,s_l^1\right)^{\alpha}}{1-\theta\left(\gamma,s_l^2\right)}\right] > 0. \tag{22}$$

Therefore, for any given $\gamma_1 \in \Gamma$, using the fundamental theorem of calculus $V\left(w_0,\gamma_1,s^0\right) = V\left(w_0,0,s^0\right) + \int_0^{\gamma_1}\frac{\partial V\left(w_0,\gamma,s^0\right)}{\partial\gamma}d\gamma > V\left(w_0,0,s^0\right)$.

4. I now focus on the additional comparative statics, still using a parametric notion of a mean-preserving spread described above. Let
$$g_0\left(x_0\right) = Af\left(\frac{x_0}{1-\theta\left(s^1\right)}\right) + x_0$$
be the function that maps $t=0$ capital expenditures, $x_0 := \wp_0 k_1$, into $t=1$ net worth. One can define its inverse
$$\phi\left(w\left(s_l^1\right)\right) = g_0^{-1}\left(w\left(s_l^1\right)\right).$$
Naturally, this represents a cost function for obtaining $w\left(s_l^1\right)$ units of net worth in state s_l^1. This function is useful for rewriting the firm's problem, using net worth at $t=1$ as the choice variables, with
$$\max_{w(s^1)}\sum_{s^1}\pi\left(s^1\right)\left[Af\left(\frac{w\left(s^1\right)}{1-\theta\left(s^2|s^1\right)}\right)+w\left(s^1\right)\right]$$
s.t.
$$w\left(s^1\right) = w\left(s_l^1\right) + \mathbf{1}_{s^1=s_h^1}\left(w\left(s_h^1\right)-w\left(s_l^1\right)\right),$$
$$w_0 = \phi\left(w\left(s_l^1\right)\right) + \pi\left(s_h^1\right)\left(w\left(s_h^1\right)-w\left(s_l^1\right)\right) \tag{23}$$
and $w\left(s_h^1\right) \geq w\left(s_l^1\right)$. Notice that $h\left(s^1\right)$ was replaced by $\left(w\left(s_h^1\right)-w\left(s_l^1\right)\right)$.
The FOCs for this problem are $w\left(s_l^1\right)$
$$\sum_{s^1}\pi\left(s^1\right)\left[Af^{'}\left(\frac{w\left(s^1\right)}{1-\theta\left(\gamma,s^2|s^1\right)}\right)\frac{1}{1-\theta\left(\gamma,s^2|s^1\right)}+1\right] = \phi^{'}\left(w\left(s_l^1\right)\right)\lambda_0,$$
with $s^2|s^1$ denoting the unique successor of $s^1 \in \left\{s_h^1,s_l^1\right\}$. For $w\left(s_h^1\right)-w\left(s_h^1\right)$,
$$\pi\left(s_h^1\right)\left[Af^{'}\left(\frac{w\left(s_h^1\right)}{1-\theta\left(\gamma,s_h^2\right)}\right)\frac{1}{1-\theta\left(\gamma,s^2\right)}+1\right] \leq \pi\left(s_h^1\right)\lambda_0. \tag{24}$$

Whenever both conditions above hold with equality, one can substitute for λ_0 and obtain, after rearranging terms, the following equilibrium condition:
$$\psi\left(w\left(s_l^1\right),w\left(s_h^1\right),\gamma\right) := \pi\left(s_l^1\right)\frac{\left[\alpha Aw\left(s_l^1\right)^{\alpha-1}\left(1-\theta\left(s_l^2\right)\right)^{-\alpha}+1\right]}{\left[\alpha Aw\left(s_h^1\right)^{\alpha-1}\left(1-\theta\left(s_h^2\right)\right)^{-\alpha}+1\right]} - \phi^{'}\left(w\left(s_l^1\right)\right) + \pi\left(s_h^1\right) = 0. \tag{25}$$

The intersection of the locus $\psi\left(w\left(s_l^1\right),w\left(s_h^1\right),\gamma\right)=0$ with the budget constraint, 23, describes the optimal choice of $\{w(s^1)\}$ in this case. The budget constraint does not depend on γ and establishes a negative relation between net worth in both states. Studying ψ, notice that its left-hand side is decreasing in $w(s_l^1)$, increasing in $w(s_h^1)$, and additionally decreasing in the spread parameter γ (through its effect on the pledgeability parameters). Therefore, an increase in γ displaces the locus defined in 25, implying that its intersection with 23 moves toward higher $w(s_h^1)$ and lower $w(s_l^1)$. Therefore, an increase in γ is associated with an unambiguous increase in $k_2(s_h^1) = \frac{w(s_h^1)}{1-\theta(s_h^2)}$ and a decrease in $k_2(s_l^1) = \frac{w(s_l^1)}{1-\theta(s_l^2)}$.

When a strict inequality holds in Equation (24), it continues to hold in an open neighborhood of γ by continuity of the objective function and constraints of the firm's problem. As a consequence, in this neighborhood, $w(s_h^1) = w(s_l^1)$, and increases in γ also increase $k_2(s_h^1) = \frac{w(s_h^1)}{1-\theta(s_h^2)}$ and decrease $k_2(s_l^1) = \frac{w(s_l^1)}{1-\theta(s_l^2)}$.

Given that firms optimally exhaust their credit capacity at $t=1$, borrowing is $b(s^1) = \theta(\gamma, s^2|s^1)\, k_2(s_h^1)$. An increase in γ leads to an increase in $\theta(\gamma, s_h^1)$ and a decrease in $\theta(\gamma, s_l^1)$. This change goes in the same direction of the changes in capital levels. Last, output at date $t=2$ is simply an increasing function of capital purchased in the previous date, so increases in the dispersion in capital purchases translate into increases in the dispersion of output.

□

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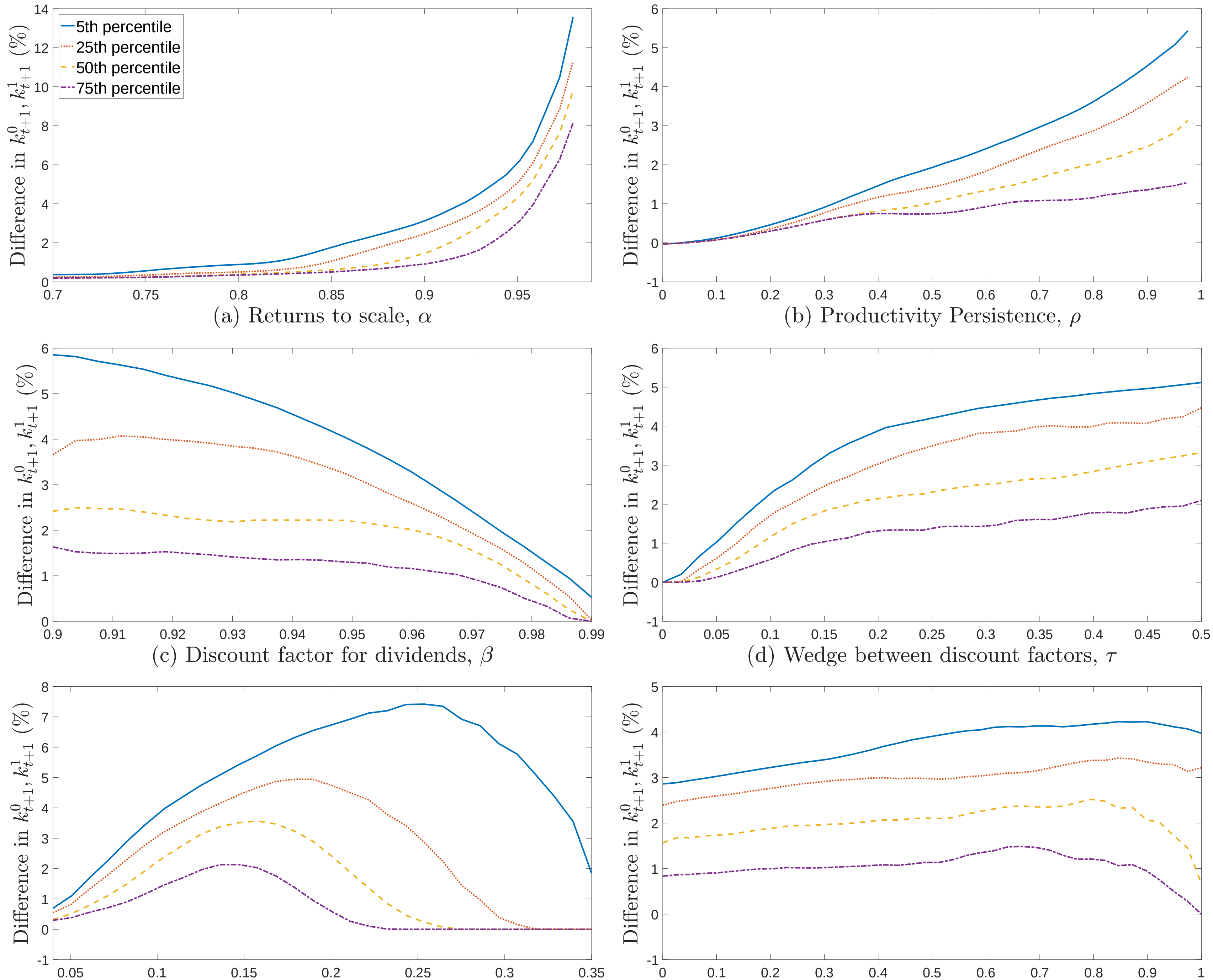
Figure 7: Comparative Statics



(a) Returns to scale, α

(b) Productivity Persistence, ρ

(c) Discount factor for dividends, β

(d) Wedge between discount factors, τ

(e) Dispersion in productivity, σ

(f) Collateralization, θ

ONLINE APPENDIX – Not for publication

OA.1 Motivating facts

In this section, I document stylized facts about cross-sectional volatility and exposure measures using Compustat data. In particular, studying sales growth (i) I confirm previous findings of a negative relationship between firm size and volatility, (ii) I show that this negative relationship accounts for more of the variation in firm-level volatility in sectors where returns on assets are more persistent, and (iii) I show that part of this volatility can be attributed to higher exposure of smaller firms to sector-level shocks. I also look at capital expenditure (CapEx) and debt growth to investigate the behavior of investment and financing and find evidence that (iv) smaller firms also display relatively more volatile capital expenditure and debt growth and (v) exposure measures of CapEx and debt growth to sector-level shocks are also decreasing in firm size.

As is standard in the literature, I restrict the sample to non-financial firms and drop regulated utilities, quasi-government firms, other non-profits, and firms reporting in currencies other than U.S. dollars. I use data starting from 1964.

First, I recreate a volatility measure, based on squared residuals of a sales growth regression like in Comin and Philippon (2005), Castro et al. (2009), and Decker et al. (2016), among others. In particular, I follow Castro et al. (2009) most closely. This regression includes controls for the year and two factors known to predict sales growth: firm age and size.[34] This squared residual measure should be interpreted as the realized conditional volatility.

I then regress this measure on two alternative size measures that serve as proxies for a firm's financial conditions: total assets and net worth. Columns 1 and 2 in Table OA.1a document that volatility shows a negative relationship with firm size, reproducing the stylized fact that smaller firms have riskier prospects. The quantitative interpretation of the coefficients is as follows. A coefficient of -0.101 in column 1 reflects the marginal impact on volatility of a one-unit increase in size (in log scale). It follows, for instance, that a 25% increase in assets decreases conditional volatility by approximately 0.025 units. This magnitude, in turn, amounts to a 10% reduction when compared to the mean squared residual (0.252) of the sales growth regression.

As suggested by Froot et al. (1993) and further developed by Rampini and Viswanathan (2013) in the context of an infinite-horizon model, the correlation between investment opportunities over time is a key determinant of endogenous hedging incentives. With persistent shocks, current high returns anticipate future high returns and present success helps fund valuable investment. On the other hand, when shocks are transitory, this is not the case, and incentives for smoothing cash flows dominate. I refine the stylized fact on cross-sectional volatility to account for a possible heterogeneity in the persistence of returns across industries.

[34] As is standard in the literature, age is proxied by the firm's first appearance in Compustat. Size is proxied by total assets. Notice that year fixed effects absorb sale growth shocks at the industry-wide level. As a consequence, squared residuals are reduced relative to regressions in which these controls are not present, like in Comin and Philippon (2005). As my interest is in the cross-sectional behavior of volatility, the distinction in levels between these approaches is not central.

Table OA.1: Cross-Sectional Volatility and Exposure Regressions

(a) Sales, CapEx and Debt Growth Volatility

	Sales Growth Vol.		CapEx Growth Vol.		Debt Growth Vol.	
	(1)	(2)	(3)	(4)	(5)	(6)
Total Asset (log)	-0.101**		-0.270**		-0.0739**	
	(0.00816)		(0.00827)		(0.00392)	
Net Worth (log)		-0.0749**		-0.242**		-0.0705**
		(0.00724)		(0.00914)		(0.00446)
Observations	225018	208404	217259	202797	231311	213134
Adjusted R^2	0.030	0.025	0.073	0.066	0.026	0.024

Standard errors in parentheses, clustered at the 3-digit SIC level. Regressions include fixed effects for 3-digit SIC industry and year.

* $p < 0.05$, ** $p < 0.01$

(b) Sales, Capital Expenditures, and Debt Growth Exposure

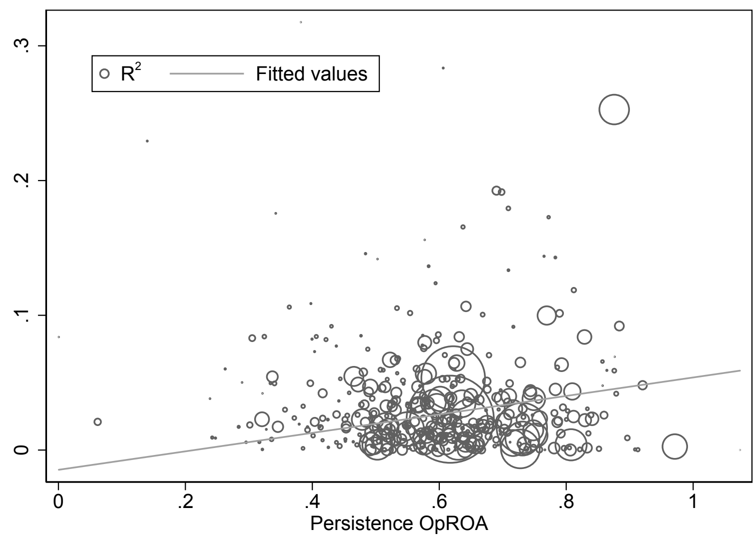
	Sales Growth		CapEx Growth		Debt Growth	
	(1)	(2)	(3)	(4)	(5)	(6)
Mean of dependent variable	0.682**	0.187	0.275**	0.182**	0.825**	0.305**
(at sector-level)	(0.0953)	(0.0968)	(0.0520)	(0.0481)	(0.0779)	(0.0611)
Total Assets (log) x Mean of	-0.0479**	-0.116**	-0.110**	-0.123**	-0.0251*	-0.0970**
dependent variable	(0.0147)	(0.0153)	(0.00818)	(0.00763)	(0.0116)	(0.00980)
Observations	225018	225018	217259	217259	231293	231293
Additional Controls	No	Age, Total Assets	No	Age, Total Assets	No	Age, Total Assets
Adjusted R^2	0.069	0.092	0.094	0.101	0.062	0.080

Standard errors in parentheses, clustered at the 4-digit SIC level. Regressions include 3-digit sector-specific intercepts and means that are computed at that level of aggregation.

* $p < 0.05$, ** $p < 0.01$

I proxy for persistence in returns in the following way. First, I compute operational returns on assets, using Compustat's operating income before depreciation (OIBDP) divided by total assets. Then I estimate a first-order autoregressive model at the 4-digit SIC level and store the autoregression coefficient as a proxy. I additionally regress realized conditional volatility (the residuals from sales growth regression) against total assets at each of these sectors. I obtain a distribution of sector-level coefficients that confirm the negative relationship between size and volatility at this level of aggregation.[35] I then ask the question: how much of the variation in the realized volatility can be accounted for by the cross-sectional dispersion in firm size? The R^2 of each of these sector-level regressions offers the answer to that question. Figure OA.1 plots that statistic against the return persistence proxy and suggests that, in sectors with more persistent returns, the negative relationship between size and volatility explains relatively more of the total variation in realized sales growth volatility.

Figure OA.1: Explained realized conditional volatility at the sector level



This evidence is suggestive, but not conclusive, of the endogenous risk-taking channel studied in the paper. Thus, I further investigate the cross-sectional relationship between size and risk exposure. For this, I regress individual sales growth against a proxy for common business conditions at the sector level. In particular, I use the mean growth in the 3-digit SIC industry as this proxy, in columns 1 and 2 of Table OA.1b, and allow for sector-specific intercepts.[36] These regressions are analogous to return regressions, with coefficients being measures of exposure to the factors on the right-hand side. Negative coefficients for the effect of the interaction (in the second row) indicate that exposure decreases with size in the cross section of firms.

The quantitative interpretation of these coefficients is as follows. I focus on the specification in column 2, which includes commonly used controls for firm growth based on age and size. The estimated effect means that a unit increase in net worth (in log scale) decreases exposure by -0.116 (so a 10%

[35]The mean coefficient is -0.077, and approximately 95% of the sector-level coefficients are negative. Figure 1 is robust to the exclusion of all positive occurrences.

[36]These results are robust to aggregation at the 2-digit SIC level.

increase decreases exposure by approximately one-tenth of that amount). To put this number into perspective, it is worth pointing out that in the absence of any controls other than sector fixed effects, the average risk exposure is mechanically unitary. In Sections 3.1 and 4 , I study how additional loading on factors that determine growth, such as demand fluctuations or input costs, can be an optimal response to the presence of binding credit constraints. In that section, when shocks are sufficiently persistent, constrained firms are shown to distort investment to favor riskier (high exposure) projects, a result that rationalizes a negative cross-sectional relationship between size and risk exposure.

While the previous literature has mostly focused on the volatility of sales growth, because of its interest in the evolution of firm size, I extend the scope of analysis to include capital investment and its financing. Columns 3–6 in Table OA.1a document that, as with sales, smaller firms also display more volatile debt growth and capital expenditures.[37] This conclusion is independent of whether one uses total assets or net worth as proxies for firm size and financial conditions.

The last four columns of Table OA.1b also extend the analysis of exposure to sectoral fluctuations to these alternative outcomes. Again, exposure to sector-wide fluctuations in both CapEx and debt growth is decreasing in firm size. Therefore, smaller firms are shown to have measures of real investment and debt growth that are both more volatile than larger firms and more exposed to common shocks at the sector level. In Section 3.2, I show that higher exposure to external credit conditions increases the value of constrained firms. Therefore, the additional volatility and exposure I observe in the data for these measures is consistent with an endogenous response to the presence of credit constraints.

OA.2 Irreversible, infrequently adjusted ("sticky"), and mutually exclusive decisions in capital budgeting

Under many circumstances, a firm faces mutually exclusive decisions, because they involve different ways of conducting a single operation. For example, an airline might expand their business class for a given route at the cost of having fewer seats available for the economy class. This is a decision that changes the behavior of its revenues, as the demands for business travel and economy-class travel have arguably different exposures to business-cycle fluctuations and also different price elasticities. Other decisions can change the type of capital goods used and hence the costs of investment and its usefulness as collateral. Although not entirely irreversible, many of these decisions might be infrequently adjustable.

I model these decisions in a general way by allowing the firm to choose an additional state variable that is carried over to the future. I denote this variable by a parameter γ from a menu Γ of exclusive alternatives, where Γ is a closed interval in $\mathbb{R}$ for simplicity. In the case of an investment scale decision in a given project, like in Myers (1974), $\gamma \in \Gamma$ denotes a previously taken decision regarding that level. Other mutually exclusive decisions, such as the allocation of previously leased aircraft and its crew to one of multiple different markets, can be represented with the same notation.

Now, the firm's problem can be solved in two steps: for each previously made decision treated as a state variable $\gamma \in \Gamma$, the optimal financial policy describing borrowing, investment, and hedging can be

[37] To put the estimates into perspective, the mean squared residual is .866 for CapEx growth and .2752 for debt growth.

obtained. Let $V\left(w,\gamma,s^{t}\right)$ denote the value achieved when decision γ is implemented and net worth is w_0. Thus,

$$V\left(w,\gamma,s^{t}\right)\equiv\sup_{d\geq0,\left\{k^{j}\right\}_{j}\geq0,\left\{b\left(s^{t+1}\right)\right\}_{s^{t+1}|s^{t}}}d+\beta E\left[V\left(w\left(s^{t+1}\right),\gamma,s^{t+1}\right)\right] \tag{OA.1}$$

subject to resource flow constraints,

$$w+R^{-1}E\left[b\left(s^{t+1}\right)\right]\geq d+\sum_{j\in J}q^{j}\left(\gamma,s^{t}\right)k^{j}, \tag{OA.2}$$

$$w\left(s^{t+1}\right)=\sum_{j\in\mathcal{J}}\left\{F^{j}\left(k_{t+1}^{j},\gamma,s^{t}\right)+q^{j}\left(\gamma,s^{t+1}\right)(1-\delta)k^{j}\right\}-b\left(s^{t+1}\right), \tag{OA.3}$$

for each $s^{t+1}|s^{t}$, as well as state-contingent collateral constraints

$$b\left(s^{t+1}\right)\leq\sum_{j\in\mathcal{J}}\theta^{j}\left(\gamma,s^{t+1}\right)q^{j}\left(\gamma,s^{t+1}\right)(1-\delta)k^{j}. \tag{OA.4}$$

Here $F^{j}\left(k^{j},\gamma,s^{t}\right)$, $q^{j}\left(\gamma,s^{t+1}\right)$, and $\theta^{j}\left(\gamma,s^{t+1}\right)$ allow for dependence of revenue functions, capital-goods price processes, and recovery rates on the decision γ. For the case in which capital levels cannot be dynamically readjusted like in the original Myers (1974) setup, one can simply add a $k^{j}=k(\gamma)$ constraint without any additional modification.

Infrequent adjustment (or stickyness, like in the literature regarding frictions in price or investment adjustment in macroeconomics) can be described through an exogenous subset of histories $S_{adj}^{t}\subset S^{t}$ at which decisions can be revised. If decision $\gamma\in\Gamma$ can be revised at node s^{t}, then the optimal decision is the solution to

$$\gamma^{*}\in\arg\max_{\gamma\in\Gamma}V\left(w,\gamma,s^{t}\right).$$

Otherwise, the state γ is carried forward to future periods. Notice that this framework can encompass both the case of a " sticky" decisions (when a random variable in the state space describes whether decisions can be revised) and the once-for-all selection of a project γ in a menu of mutually exclusive alternatives Γ (i.e., the limit at which no decision can be revised after the initial node s_0).

I will proceed by characterizing the consequences of local changes in γ. These are more tractable and the properties of large changes can be understood as the integration of marginal changes. Therefore, I locally evaluate the combined impact of changes in the cash-flow process, the expenditures on capital goods, and different recovery rates induced by a potential change in a partially irreversible decision. The final impact of a different decision on the firm's value is a composition of the effects through these three channels.

Toward that end, I define the net investment at s^{t} in the standard way as $i^{j}\left(s^{t}\right)\equiv k^{j}\left(s^{t}\right)-(1-\delta)k_{t}^{j}\left(s^{t-1}\right)$, where $k^{j}\left(s^{t}\right)$ is used in production at $t+1$. Additionally, I write the endogenous borrowing capacity from s^{t+1} toward its predecessor s^{t} as

$$BC\left(\left\{k^{j}\right\}_{j},\gamma,s^{t+1}\right)\equiv\sum_{j}\theta^{j}\left(\gamma,s^{t+1}\right)(1-\delta)q^{j}\left(\gamma,s^{t+1}\right)k^{j}\left(s^{t}\right), \tag{OA.5}$$

and the firm's net cash flow as

$$NCF\left(\left\{k^j\left(s^t\right), i^j\left(s^t\right)\right\}_j, \gamma, s^t\right) \equiv \sum_j \left[F^j\left(k^j, \gamma, s^t\right) - q^j\left(\gamma, s^t\right) i^j\left(s^t\right)\right]. \tag{OA.6}$$

Given those two definitions, one can study the consequences of a project selection decision, which changes both cash flows and the firm's financing conditions. Expression OA.7 can be seen as a generalization of Equation (15) for the case of infrequently adjusted decisions and mutually exclusive projects.

Proposition 3. *The shadow value of a marginal change in the decision $\gamma \in \Gamma$ is given by*

$$\frac{\partial V\left(w, \gamma, s^t\right)}{\partial \gamma} = E_t\left[\sum_{h=t}^{\tilde{T}} \beta^{h-t} \frac{\partial V\left(w^*\left(s^h\right), \gamma, s^h\right)}{\partial w\left(s^h\right)}\left(\frac{\partial NCF\left(k_h^*, i_h^*, \gamma, s^h\right)}{\partial \gamma} + \mu\left(s^h\right) \frac{\partial BC\left(k_h^*, \gamma, s^h\right)}{\partial \gamma}\right)\right], \tag{OA.7}$$

where $\mu\left(s^h\right) \equiv \frac{\partial V\left(w\left(s^{h-1}\right), s^{h-1}\right)/\partial w\left(s^{h-1}\right)}{\beta R\left(\partial V\left(w\left(s^h\right), s^h\right)/\partial w\left(s^h\right)\right)} - 1 \geq 0$ is a normalization of the multiplier on the collateral constraint that limits borrowing between s^{h-1} and s^h and $\tilde{T} \leq \infty$ is the (potentially random) variable describing the next decision revision time.

Proof of Proposition 3. Marginal changes in γ lead to changes in cash flows, prices, and recovery rates; each of these contributes to part of the marginal effects into the value function at s^t. I use the envelope theorem recursively, until a node in which γ can be reoptimized is reached. After some algebra, the terms involved are represented by $E\left[\sum_{h=t+1}^{\tilde{T}} \beta^{h-t} \frac{\partial V\left(w\left(s^h\right), \gamma, s^h\right)}{\partial w\left(s^h\right)} \sum_j \frac{\partial F^j\left(k_h^*, \gamma, s^h\right)}{\partial \gamma}\right]$ for the discounted value of changes in cash flows, $\beta^{h-t} \pi\left(s^h | s^t\right) \frac{\partial V\left(w, \gamma, s^h\right)}{\partial w\left(s^h\right)}\left\{-i^{j,*}\left(s^h\right) + \mu\left(s^h\right)(1-\delta) \theta^j\left(\gamma, s^h\right) k_h^{j,*}\left(s^{h-1}\right)\right\}$ for a change in each capital good price, and $\beta^{h-t} \pi\left(s^h | s^t\right) \frac{\partial V\left(w\left(s^h\right), \gamma, s^h\right)}{\partial w\left(s^h\right)} \mu\left(s^h\right) q^j\left(\gamma, s^h\right)(1-\delta) k_h^{j,*}\left(s^{h-1}\right)$ for a change in the recovery rate. Combining these terms and using definitions OA.5 and OA.6 for borrowing capacity and net cash flows, one obtains expression OA.7. □

Returning to the firm's choice among all possible alternatives, I can also use Equation (OA.7) to describe the firm's project selection in the following way. Any interior solution needs to satisfy $\frac{\partial V\left(w, \gamma, s^t\right)}{\partial \gamma} = 0$. This also becomes a sufficient condition for an interior optimum whenever (OA.5) and (OA.6) define net cash flows and borrowing capacity that are concave in γ. In this case, one can think in terms of a firm that takes as given the value of internal funds and the premium on borrowing capacity obtained from the operation of the optimal project, and acts as if maximizing the sum of discounted cash flows plus the premium-adjusted borrowing capacity.

In more general cases, a root in Equation (OA.7) is a necessary condition for an interior optimum, allowing one to rule out projects that can never be optimal. It also serves to illustrate how different decisions, such as favoring a project with marginally riskier or safer cash flows, can influence the firm's value.

OA.3 Constraints to risk management: Incomplete instruments

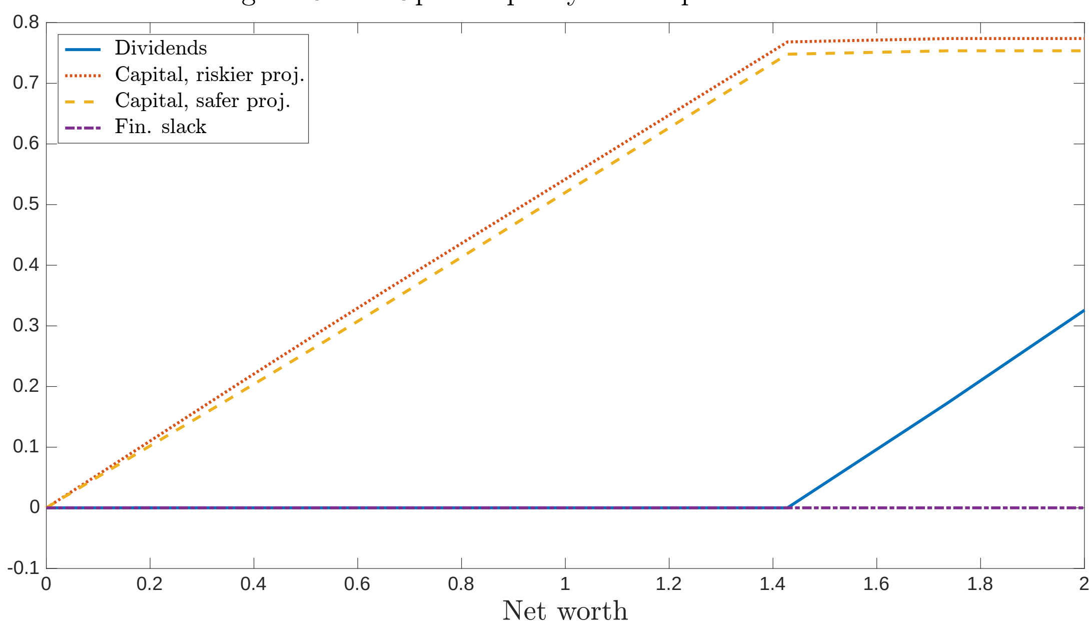
In this section, I illustrate how less flexibility in financial risk management translates into increased distortions in capital budgeting.

Although firms have access to many possible contractual contingencies, renegotiation possibilities, and financial instruments, one could reasonably argue that a benchmark that allows full contingency in traded securities or designed contracts overstates actual risk management possibilities. Indeed, this is the main motivation behind two parallel strands of literature in both corporate finance and macroeconomics that impose some additional form of market incompleteness.

To study the the impact of additional restrictions on financial flexibility, I slightly modify the setup studied in the body of the paper. The modification lies in forcing collateralized debt to be riskless, that is, formally, $b\left(s^{t+1}\right)$ is required to be measurable with respect to information available at date t. In this modified version of the model where financial instruments are incomplete, capital investment, the accumulation and liquidation of liquidity buffers, and dividend policies offer some important flexibility.

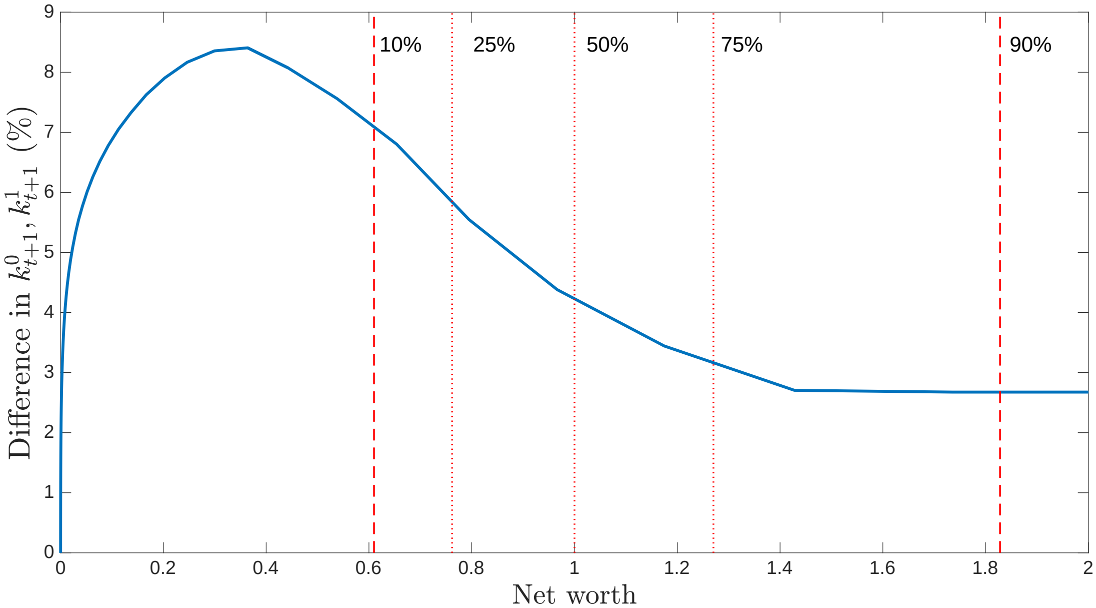
I plot the optimal policies (Figure OA.2) and the distortions in capital budgeting along the stationary distribution (Figure OA.3) using the same parameter values from the previous example. The median net worth is essentially the same as that in the complete instruments benchmark; it displays a 2% reduction.

Figure OA.2: Optimal policy: Incomplete instruments



This figure illustrates the optimal policy as a function of current net worth in the presence of incomplete set of financial instruments for risk management.
The parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, and $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds is $R = 1.04$. The wedge, $\tau = 0.2$, between the discount factor and the cost of capital is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution, and financial variables are normalized by the first-best level of capital.

Figure OA.3: Distortion in capital allocation: Incomplete instruments



This figure illustrates the percentage distortion in capital allocation across the two projects of different risk exposure along the stationary distribution in the presence of an incomplete set of financial instruments for risk management. The parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q^j_t = 1$, $\theta = 0.493$, and $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds is $R = 1.04$. The wedge, $\tau = 0.2$, between the discount factor and the cost of capital is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution.

Interestingly, all firms in the intermediate productivity state of the Markov chain studied find themselves against their borrowing constraints. It is only high net worth, less productive firms (in the lowest productivity state) that keep any liquidity buffers or unused debt capacity. This particular feature of the three-state approximation to the productivity process vanishes for finer approximations and some alternative parameter values.

Despite the hedging motive not being strong enough to make firms in this intermediate state leave financial slack, it induces distortions in capital budgeting that are from 50% to 200% higher in magnitude than the distortions induced when financial risk management is complete. This distortion still drops as net worth becomes larger, but stabilizes at 2.68% instead of 0.66% for sufficiently large firms.

This example serves as an illustration that as alternative financial instruments become less accessible, distortions in capital budgeting become significantly larger and also potentially more noticeable across the whole cross section of firms, as opposed to a phenomenon particularly concentrated on small firms.

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